

Abstract

The study deals with explanation and justification in Israeli mathematics textbooks and with the textbook's contribution to shaping their learning in the classroom. The study comprises two parts. The first part examines the opportunities offered in 7th grade Israeli textbooks for students to learn how to explain and justify mathematical statements. The second examines the ways in which the textbook, in conjunction with the teacher and the students, shape the opportunities offered in 7th grade Israeli classrooms to learn how to explain and justify.

Part I of the study investigates the explanations and justifications offered in 7th grade mathematics textbooks and the *paths of justification* – i.e., the sequences of justifications each textbook offers for each mathematical statement. The data sources include the textbook chapters introducing ten key mathematical statements, in eight Israeli 7th grade mathematics textbooks (two of limited scope, intended for students with low achievements; six of standard/expanded scope, intended for the general student population). Comparative analyses of the paths of justification, by textbook and by mathematical statement, focused on three attributes: (1) path length – the number of instances of justification offered in a textbook for each mathematical statement; (2) characteristics of the instances comprising each path – *justification types* (a meta-level characteristic, following Stacey & Vincent, 2009) and *justification strategies* (a content-specific characteristic, dealing with the specific warrant); and (3) sequencing – the order in which justifications were offered in the textbook.

Part II of the study investigates the ways in which textbooks, in conjunction with the teacher and the students, shape the opportunities offered in 7th grade mathematics classrooms. This part of the study revolves around two case studies, each focused on a mathematics teacher who uses the same textbook in two 7th grade classes. The data sources include 11-14 lesson observations in each of the four classes (a total of 49 lessons) and teacher interviews. Comparative analyses of the paths of justification, by teacher, by classroom, and by topic, focused on three attributes: (1) path length, (2) characteristics, and (3) sequencing.

The findings of Part I reveal that the analyzed textbooks provided justification for all analyzed statements (all but one statement in one textbook). Path lengths varied considerably – where some textbooks offered long paths for a mathematical statement, other textbooks offered rather short paths. Paths of justification typically comprised either deductive or empirical justification types, whereas external types were extremely rare.

Three justification types were especially common – an empirical type (*Experimental demonstration*) and two deductive types (*Deduction using a specific/general case*). However, these types were distributed differently among mathematical topics and among textbooks of different scopes: (1) *Deduction using a general case*, the justification type closest to a formal proof, was included solely in geometry, while *Deduction using a specific case* (i.e., generic examples) was used mostly in statements relying on an algebraic derivation; and (2) *Deduction using a general case* was roughly three times more common in textbooks of standard/expanded scope compared with textbooks of limited scope. The three most common justification types were often similarly sequenced across textbooks and topics: paths that involved both experimentation and a deductive process tended to offer the empirical type before the deductive, and paths that involved deduction using both a generic example and the general case tended to offer the specific case before the general.

Justification strategies were associated with justification types, yet the correspondence was not one-to-one. In algebra statements, almost every justification strategy corresponded to a single type across textbooks. In geometry, however, justification strategies often occurred several times in paths of justification, with various justification types.

The findings of Part II suggest that the textbook contributed greatly to shaping the paths of justification and was the main source for justifications in all observed classrooms. Paths of justification in every class were generally similar to, yet typically shorter than, the paths offered in the textbook – both in their characteristics and in their sequencing.

Additionally, the findings reveal that the teachers' perception of their students' abilities was instrumental in constructing the paths of justifications in the classes. Instances of justification were excluded if the teacher regarded them as too difficult in two cases: (1) the justification type was *deduction using a general case*, and (2) the textbook marked them as intended for high-achieving students. Additionally, noisy classroom environment interrupted several discussions and contributed to altering a justification type from deductive to empirical.

This study focuses on textbooks that are currently in use and brings to light certain nontrivial aspects: It maps the paths of justification offered for mathematical statements and characterizes both the justification strategies and the corresponding justification types. Additionally, the study discusses the contribution of the textbook to shaping classroom learning of explanation and justification. This information is important for researchers, educators, textbook authors, curriculum developers, and decision makers.