

Argumentation and school mathematics

Thesis for the degree DOCTOR OF PHILOSOPHY

Submitted to the Scientific Council of the Weizmann Institute of Science

Rehovot, Israel

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2011

Abstract

This research deals with argumentation and school mathematics. It consists of two parts. Part I investigates views of mathematics educators on deductive reasoning and on the commonly stated goal of using mathematics learning to develop deductive reasoning that is usable outside of mathematical contexts. The data source includes 21 individual semi-structured interviews. The findings of the study show that the interviewees ascribed different meanings to the above mentioned goal. Moreover, none of them said that it is possible to develop formal logic-based reasoning useful outside of mathematics, whether because they thought it is impossible or because they referred to other characteristics of deductive reasoning that can be developed: general argumentative skills (e.g., constructing and evaluating arguments) or systematic aspects of reasoning. Three distinct views were identified: the intervention–argumentation view, the reservation–deductive view, and the spontaneity–systematic view. Each interviewee’s view was interrelated with his/her approach to deductive reasoning and its nature in mathematics and outside it.

Part II of the research evolves from the findings of Part I: A large group of interviewees claimed that learning mathematics can develop argumentative habits of mind, provided there is support from the teacher and the curriculum. Part II examines, in the context of transformational algebraic activities, the opportunities for class argumentation provided by two teachers (Sarah and Rebecca), each holding a different view identified in Part I of the research, and the ways in which these opportunities were shaped by the mathematical situation and the classroom. Each teacher taught two classes, using the same innovative curriculum materials. The study focuses on 15 units on the topic equivalent algebraic expressions, which is the starting point for algebra in the curriculum program.

As a first step, this part of the study examined the transformation-related (TR) ideas addressed in the four classes. The analysis revealed several TR ideas,

classified into two conceptually different approaches to transformational algebraic activities: TR ideas associated with developing meaning for algebraic expressions, and TR ideas associated with developing understanding of equivalence of expressions. Whereas similarities were found in all classes with regard to TR ideas associated with developing meaning for algebraic expressions, differences were found between teachers and between classes taught by the same teacher with regard to TR ideas associated with developing understanding of equivalence of expressions. The kinds of reasoning required (i.e., inductive reasoning vs. deductive reasoning) appear to account for some of these similarities and differences as well as the teachers' classroom practices regarding the nature of students' participation in the class discourse.

The second step was to examine the argumentation encouraged in the classes. Based on the findings of the first step, the second one focused on four central units, two of which encourage inductive reasoning, while the other two encourage deductive reasoning. Analysis of the teachers' and the students' argumentative moves in the whole-class activities revealed a unique argumentation approach typical of each teacher, which was manifested in both her classes in the two couples of units. In Sarah's approach to argumentation, students were exposed to mathematical arguments and to explicit ideas regarding proving; however, this approach did not allocate a significant role to students in their generation and evaluation. Rebecca's approach to argumentation largely shifted the responsibility for justifying and evaluating claims to the students, but at the same time seldom encouraged discussions of the arguments or suggested explications of the argumentation that occurred in class. The analysis also showed, for each teacher, a certain strengthening of her approach in a classroom perceived by her as having more difficulties. Moreover, whereas a similarity was found in the argumentative occurrences between Sarah's classes, some differences were found between Rebecca's classes. These differences were expressed in different types of justifications provided by students (based on mathematical rules vs. numerical examples) and in the extent to which dialectical discourse developed in each class. The intersection of mathematical situations that involved deductive reasoning and Rebecca's approach to argumentation along with its interaction with the students in each class appear to account for these differences.

The findings indicate a certain consistency between each teacher's approach during the interviews (in Part I of the research) and her actual teaching. Rebecca's statement that mathematics teaching should be oriented toward developing argumentative habits of minds was indeed expressed in her lessons, although not to the full extent. Sarah's statement that learning mathematics spontaneously develops systematic habits of mind, without a

need for directive intervention, was expressed in her systematic and orderly presentation of the mathematical ideas and in the limited emphasis she put on involving students in the argumentative activity.