Knowledge Development of a Pair of Students: Beginning Algebra in an Interactive Environment

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Abstract

This study traces patterns of changes taking place among a pair of students while learning a beginner's algebra course in the seventh grade with the mediation of a spreadsheet. These patterns concern relations between cognitive and social processes.

There are different approaches to the teaching and learning of Algebra. Bednarz, Kieran and Lee (1996) overview what they call the "approaches for a significant teaching of algebra". One approach consists of generalizing patterns (geometrical or numerical), and of finding rules and connections among numerical phenomenon. A different approach consists of a systematic program of problem solving. A third approach concerns modeling physical phenomena, and a fourth approach focuses on the concept of variable and function. Moreover, the presence of the computer in school and in the mathematics classroom has raised a variety of new ideas regarding the inclusion of the computer in learning mathematics. During the last decade several attempts were made to integrate computerized tools for teaching and learning mathematics in general, and particularly in teaching algebra (Heid, 1995; Hershkowitz et al., in press; Yerushalmy & Schwartz, 1993; Kieran, 1992).

A curriculum for an introductory course in algebra based on the use of a spreadsheet program has been developed at the Weizmann institute of science, as part of the CompMath project. The approach adopted by the development team is a function approach to algebra. The activities are based on the construction of algebraic generalizations of patterns for phenomena of change mainly in the context of problem situations, and in the context of visual patterns. The design of the course included the development of an entire learning environment: a series of activities for the classroom, ways of teaching, social organization in the class, the role of the teacher, assessment and so on. The theoretical tenet that governed the development of the instruction was a socio-cultural perspective of human development. In other words, it was taken for granted that students build their knowledge while interacting with their environment, in different social settings – with one peer, within a small group of students, with the teacher, with the whole class. The technological tool – the Excel program, which has been recognized as a powerful tool for learning mathematics, is available for the use of the students.

The teacher's role consists of organizing and guiding the work in groups, and of managing the whole class discussion.

In order to trace changes along a full academic year in students' ways of working from both a cognitive and an interactive-social perspective, five activities that were chosen could be compared regarding their mathematical content and their structure. All students worked in dyads in the computer laboratory. It was then natural to focus on one pair of students. The pair was selected according to their good verbal propensity. All activities of the pair were videotaped, and written reports were collected too.
The general research question raised was: how does the mathematical knowledge of a pair of students, participating in an introductory course in algebra, mediated by a spreadsheet program, change along activities during one year?

This general question was instanced in four research questions:

1. How do the processes of conjecturing and generalizing evolve along the course?
2. How do the patterns of the interaction modify along the course?
3. How is knowledge constructed in an activity consolidated, i.e. how it is used in later activities?
4. What is the contribution of each of the students to the construction of the shared knowledge?

The data collected from the dyad was analyzed in two ways. First, each activity was analyzed according to three perspectives: cognitive, interactive, and integrative. The cognitive perspective was conducted according to the method of verbal analysis (Chi,1997). The interactive analysis followed Resnick et. al. method of analysis of conversation (1993). Finally, in each activity I tried to integrate cognitive and interactive perspectives. The second way of analysis compared the analyses of similar activities along the year. While the selection of protocols compared was a priori based on the comparison of cognitive constructs, the analyses integrated the changes that occurred in interactive processes.

The main findings in this study are:

- First, the description of knowledge construction along the year as a socio-cultural process (first and second research question): It is shown that the pair collaborates by sharing a common motive. During interactions, each student contributes by explaining ideas to their peer, by agreeing or disagreeing with suggestions of the peer. Moreover, students by turns appropriate or diffuse knowledge, or jointly construct knowledge. The interaction of the pair with the computer helped in the realization of varied goals: displaying phenomenon in numerical and graphical representations; checking hypotheses in numerical, graphical and algebraic representations, and deciding between conflicting opinions.

- The ability to hypothesize developed (first research question). It was found that hypothesis that involves the comparison between similar phenomena (relative hypothesis) develops first. It is harder to hypothesize what will happen at a given time for a given phenomenon of growth (absolute hypothesis).
- Construction of knowledge and its consolidation were seen (third research question). Several types were identified. In one case, knowledge ("right" or "wrong") was constructed quite quickly during on task, and consolidated within the same activity, as it was used in following tasks. In another case, evidence was found for construction of knowledge in an ongoing dialectical process, between construction and consolidation, which took place between several activities, when pieces of knowledge incrementally cumulate from one activity to another one.
- The ability to construct generalizations developed in two senses (first research question). First, generalization was attained through recursion. The mediating role of the Excel program (especially the "dragging" operation) is central in the development of this way to generalize. Secondly, generalization is attained through the elaboration of an explicit algebraic expression with the place number. At the beginning of the year, students construct recursive generalizations easily, by using the computer. The students do not feel the need to generalize with an explicit expression at that time. Later on during the year, the need for finding an explicit expression grows among the students, and this need leads them to generalize explicitly.
- Patterns of interaction between peers change rapidly during activities as well as between them (second research question). The type of interaction is connected to the cognitive difficulty of the task at hand, and the previous knowledge of each of the peers. This flexibility shows that the students use the mutual resources of the pair correctly.

Practical applications that emerge from this study:
This study was conducted in a learning environment that was developed as part of the CompuMath project. Some of the basic characteristics of the introductory course, which was reflected in the five activities, were checked in this study. Here are some points that emerge in that connection:

- The structure of the activities gives opportunities to foster inquiry about mathematical processes, such as hypothesizing and generalizing. As was found, those abilities developed during the course.
- The ability to hypothesize relative hypothesis develops before the ability to hypothesize absolute hypothesis. Therefore, the design of

activities as a continuum for a whole course should include opportunities to hypothesize relative hypotheses first, then in later activities, absolute hypotheses.

- The processes of generalization emerging from growth phenomenon were different from those emerging from the identification of visual patterns in a given sequence. Therefore, it is important to include activities of both kinds in the course.
- One of the key ideas in the design of the course was that the student should meet the same mathematical content and concepts in different occasions, in a spiral way. And indeed, this idea leaded the students to construction of knowledge. The new "meeting" with the same phenomenon should include similar and different characteristics hopefully leading to dialectical construction of knowledge.
- As students tend to generalize first through recursion, presenting them with non-consecutive elements of a sequence pushes them towards explicit generalization.
- Working with the same student along the year appears to be beneficial for the dyad. Therefore, design of activities should take into account how to create opportunities for the most beneficial effects of interaction.
- The technological tool played different roles: it allowed (1) experiencing various phenomena, (2) verification of challenge for numerical or graphical hypotheses, (3) dynamic manipulations of the data (in order to reach a better understanding of a phenomenon), (4) checking the validity of an algebraic generalization, (5) and arbitration in case of disagreement. Moreover, students used the tool wisely, meaning that they acted with the tool for various reasons, to create appropriate new meanings. It is the curse designer's responsibility to include various activities that create the opportunities to use the computerized tool in different ways.

Possible directions for further research

Once the questions set could be answered, the study opened new questions on construction of knowledge in a collaborative mode.

i. Are interaction patterns determined by cognitive aspects, and how?

It was found that in different parts of the collaboration the patterns of interaction changed and that the partner who is more knowledgeable in the topic at hand takes the lead. When both students are knowledgeable, they both contribute (differently!) to the learning process. When neither of them is knowledgeable, each of them tries to develop ideas separately. I do not know whether such connections between cognition and interaction processes are typical or idiosyncratic.

ii. What is the knowledge left in the individual after collaborative work?

Students who work together construct their knowledge while collaborating. However, the learning group is composed of individuals. The shared knowledge is reflected in the knowledge of the individuals who composed the group. The knowledge, which was constructed by the individual, is the basis for additional individual and/or common learning. Therefore, it is important to trace such processes of knowledge construction. There is a methodological dilemma concerning the ways of tracing these processes (Hershkowitz, 1999). Kieran tries to answer that question, by giving a post-test to individuals after the joint work as pairs (Kieran, 1999). The question is, whether this is an appropriate methodology, and if not, what is the alternative? In the present study, I tried to find evidence for the consolidation of knowledge in an activity, by tracing the use of that knowledge in a later activity.

iii. How can consolidation of constructed knowledge by students be encouraged?

As was shown in this study, the process of consolidation can take many different faces; in one case, knowledge ("right" or "wrong") was constructed quite quickly in one task, and evidences for its consolidation were found within the same activity, and in the following activities. In another case, evidence was found for construction and consolidation of knowledge in an ongoing dialectical process, which took place along several activities, when pieces of knowledge incrementally cumulate from one activity to the other. What are the factors that brought the student to construct their knowledge and to consolidate it? How can we encourage these construction and consolidation of knowledge processes? What is the right research methodology that allows us to follow consolidation? All these questions will need more research.