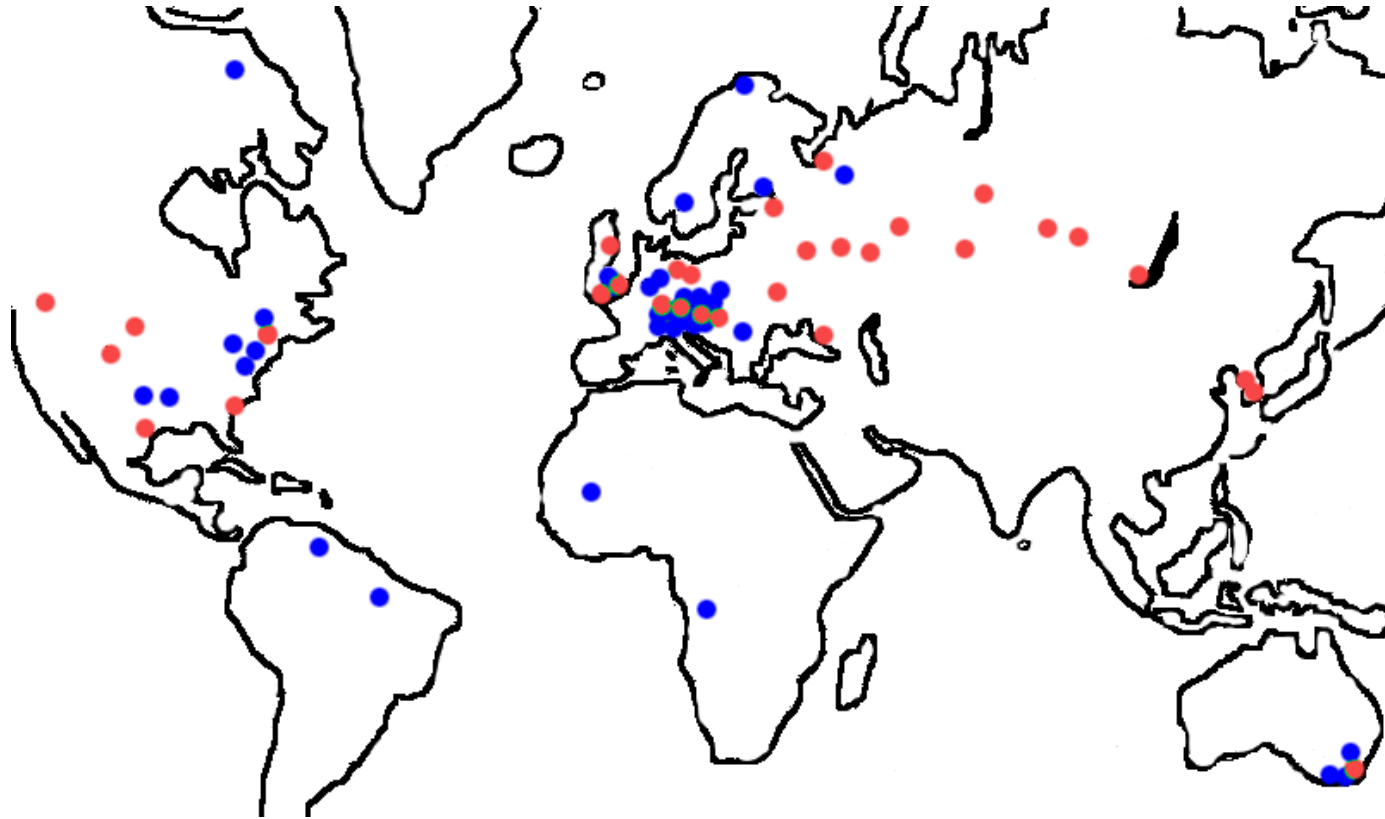


Universal scaling in precipitation and river flows

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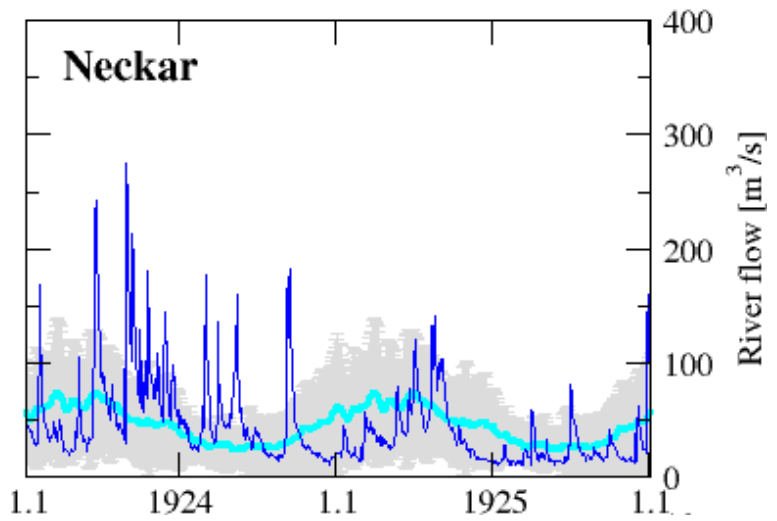
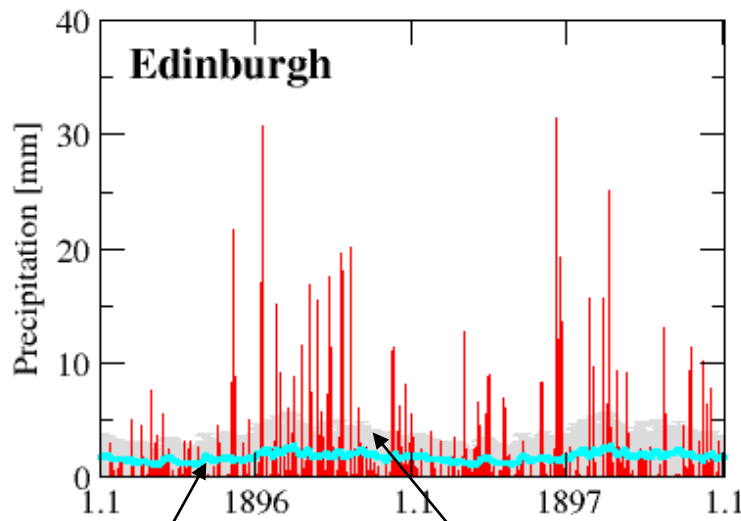
Can the complex interplay between linear and nonlinear correlations in precipitation and river flows give rise to universal behavior?



River flows: 25 records. The basin areas vary between 390 and 613,830 km², different climate zones, different soil conditions

Precipitation: 32 records. Different climate zones

Precipitation and river flows



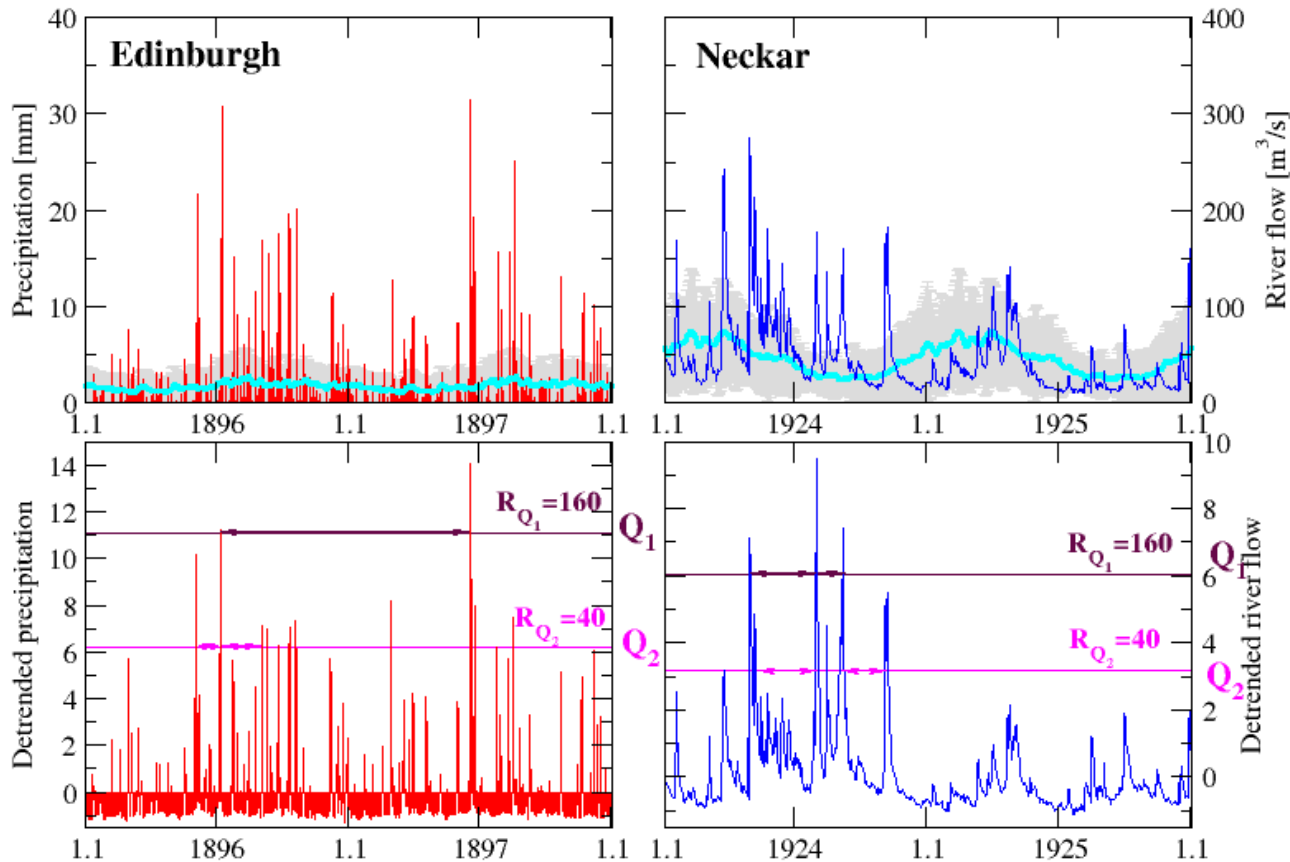
Seasonal trend

Seasonal standard deviation

Standardizing by seasonal detrending

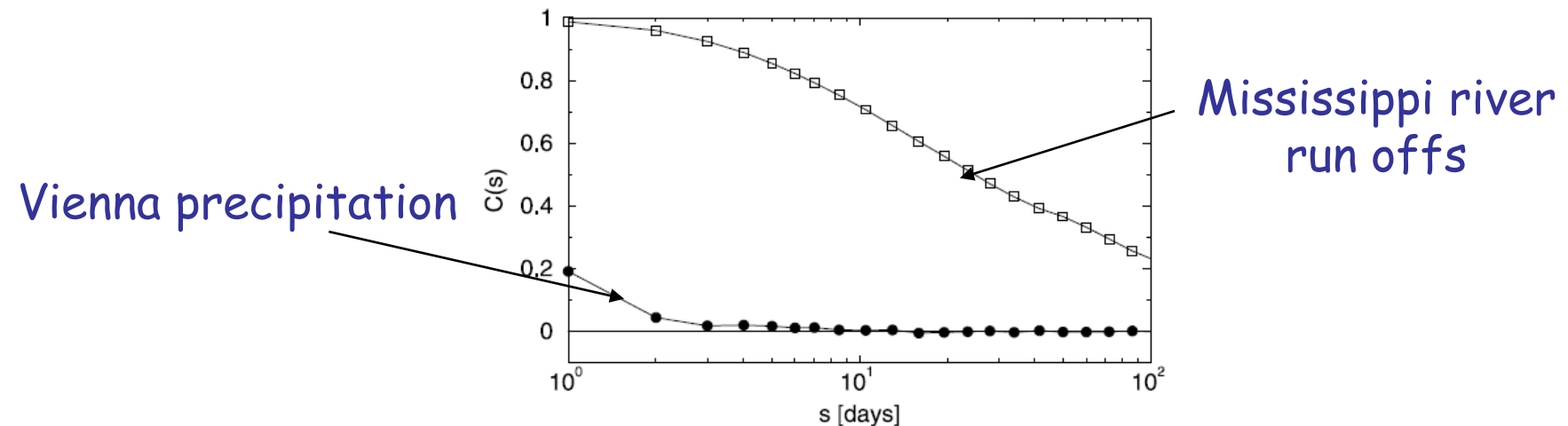
$$\tau_i = \frac{x_i - \langle x_i \rangle}{\sigma_i}$$

$\langle x_i \rangle \leftarrow$ Seasonal mean
 $\sigma_i \leftarrow$ Seasonal standard deviation



Persistence in Precipitation and River Flows

$$C(s) \equiv \langle \tau_i \tau_{i+s} \rangle = \frac{1}{N-s} \sum_{i=1}^{N-s} \tau_i \tau_{i+s} \sim (1-\gamma)(1-\gamma/2) s^{-\gamma}$$



Vienna precipitation: Short-term persistence $\gamma \approx 1$

Mississippi river flows: Long-term persistence $\gamma \approx 0.4$

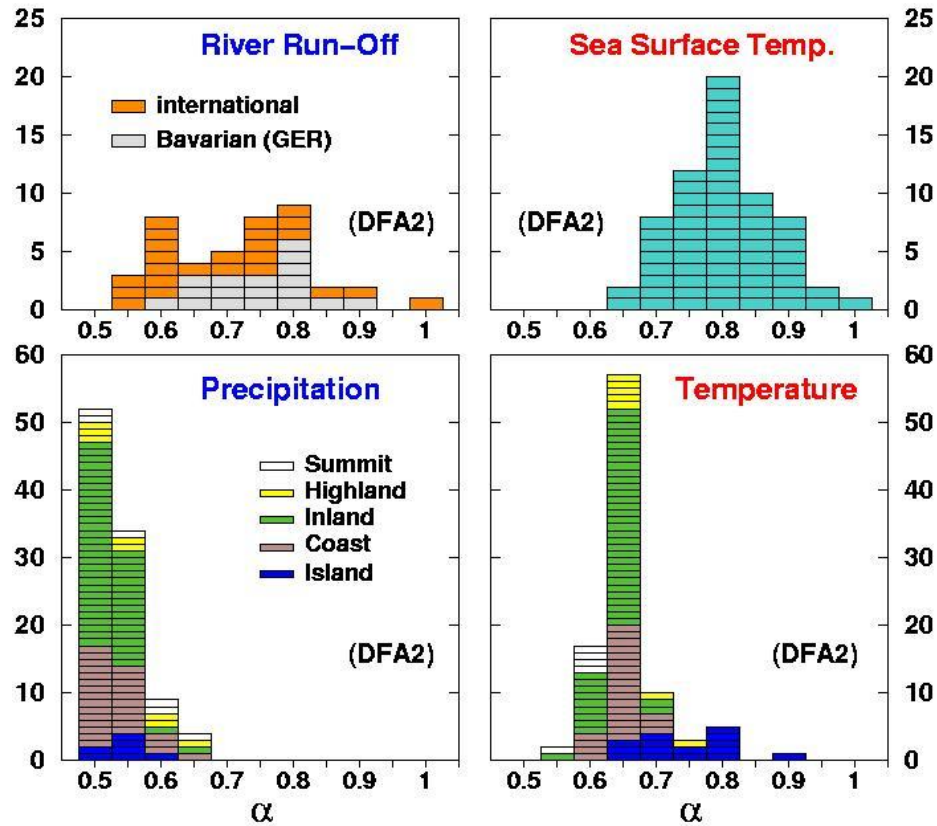
Alternative method: Fluctuation function

$$F(s) \equiv F_2(s) \equiv \left\langle \left| \sum_{i=1}^s \tau_i \right|^2 \right\rangle^{1/2} \sim s^\alpha, \quad \alpha = 1 - \gamma/2$$

Advantage: Modifications allow to eliminate systematically polynomial trends and to distinguish between short and long-term correlations :

(i) Wavelet methods WT0, WT1, WT2,...

(ii) Detrended Fluctuation Analysis: DFA0, DFA1, DFA2



$$\alpha = 1 - \gamma / 2$$

Multifractality: Generalized fluctuation function

$$F_q(s) \equiv \left\langle \left| \sum_{i=1}^s \tau_i \right|^q \right\rangle^{1/q} \sim s^{\alpha(q)} \quad \alpha(2) = 1 - \gamma/2$$

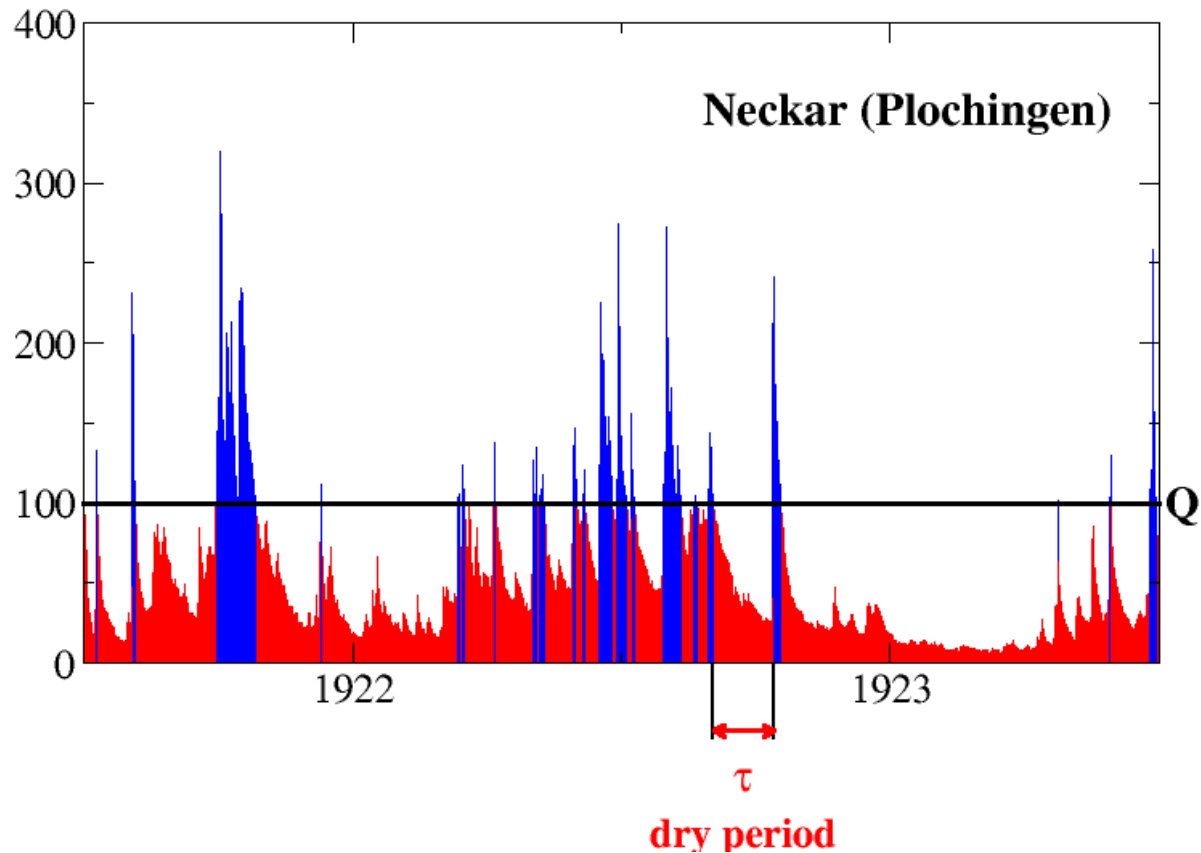
Monofractal: $\alpha(q) = \alpha$ independent of q

Multifractal: $\alpha(q)$ dependent on q

Precipitation and river flows exhibit multifractality,

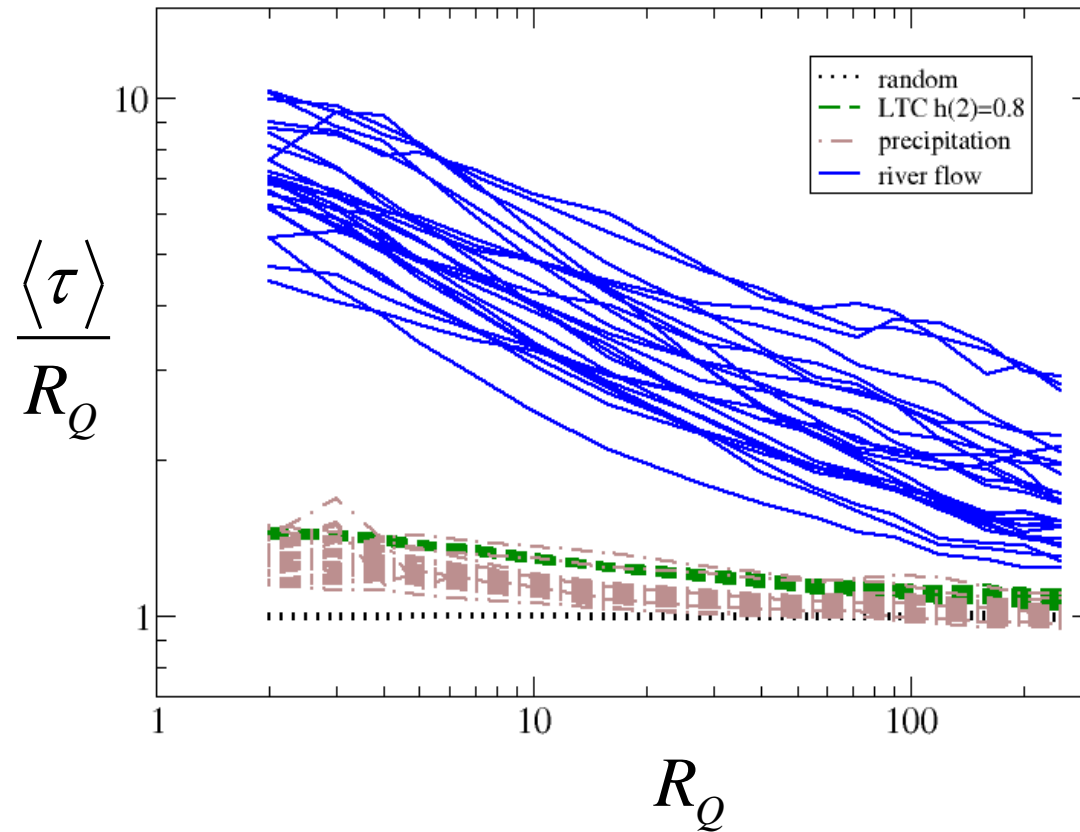
with nonuniversal exponents $\alpha(q)$

Searching for universal behavior: **Low** water periods

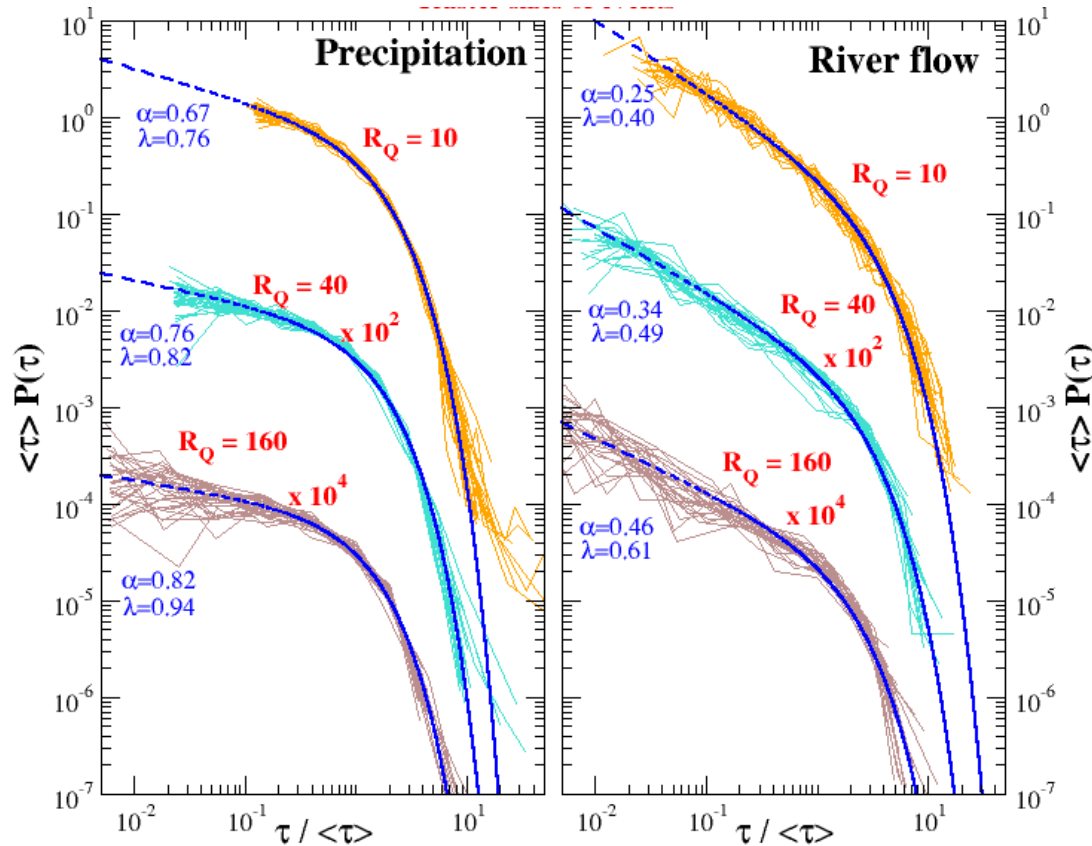


$Q \Leftrightarrow R_Q \leftarrow$ Mean return time between events above Q

Mean duration of dry periods in precipitation and river flows



Universal scaling



Functional forms of the PDFs are described by **GAMMA-distributions:**

$$P(r) \propto r^{\alpha-1} e^{-\lambda r}$$

Scaling function depends on threshold:

Signature of **MULTIFRACTALITY**