Energy Transfer by Inertial waves in Rotating Turbulence

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The context: Want to understand turbulence in 3D rotating systems.
Atmosphere, Oceans, Flows within the Earth's mantle other planetary flows.

Students involved:
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Sergio Lupo

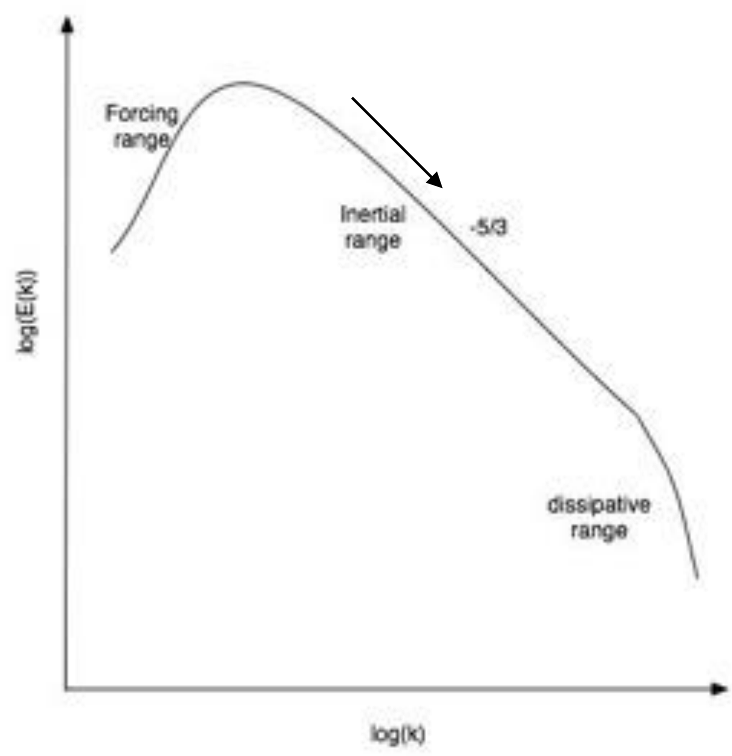
Outline

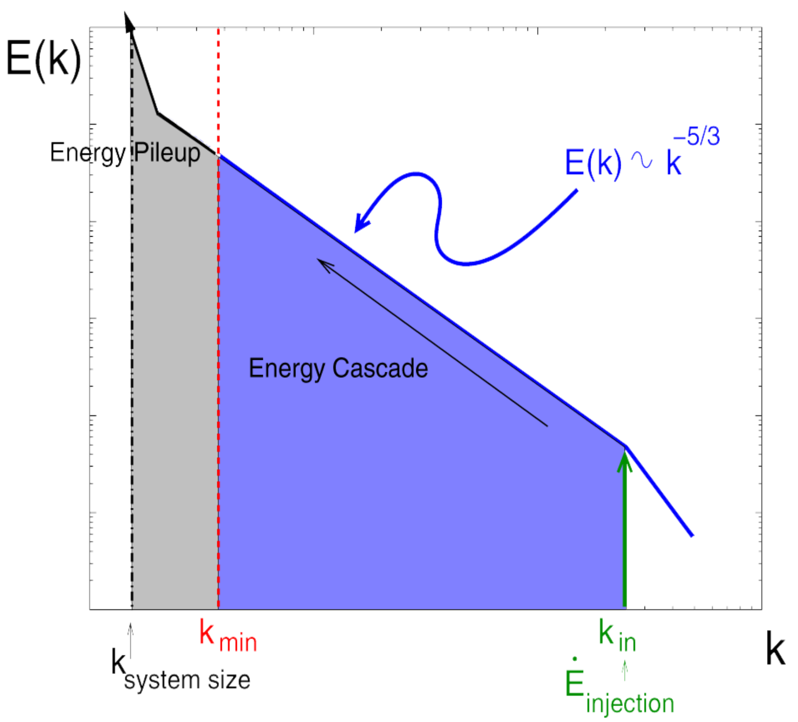
- The connection between rotating and 2D turbulence
- Inertial waves in rotating fluid
- Energy transfer in rotating turbulence
- Statistics
- A call for wave turbulence description

What is the right framework to describe 3D rotating turbulence?

3D isotropic turbulence – Richardson, Klmogorov, forward energy cascade…

2D isotropic turbulence – Batchelore, Kraichnan, inverse energy cascade…





Rotating 3D turbulence is often described in terms of 2D turbulence

A central question:

The equivalence and differences between rotating and 2D turbulence

The suggestion for equivalence is sometimes justified using **Taylor-Proudman's theorem**

$$\cancel{\frac{\partial \vec{u}}{\partial t}} + \cancel{\vec{u}\nabla}\vec{u} = -\frac{\nabla p}{\rho} + \cancel{\nu\nabla^2}\vec{u} - 2\vec{\Omega}\times\vec{u} \qquad \Rightarrow \vec{\Omega}\cdot\nabla\vec{u} = 0$$

For $\vec{\Omega} = \Omega_z$ We have: $$\frac{\partial}{\partial z}\vec{u} = 0$$

Quasi 2D

Another question:

What is the mechanism that maintains two dimensionality of the flow?
(How energy and momentum are transferred along the axis of rotation?)

$$\frac{\partial \vec{u}}{\partial t} + \cancel{\vec{u}\nabla}\vec{u} = -\frac{\nabla p}{\rho} + \cancel{\nu\nabla^2}\vec{u} - 2\vec{\Omega}\times\vec{u}$$

For small and slow perturbations to fluid at rest (in the rotating system)

Coriolis driven inertial waves propagate along the axis of rotation (Greenspan 1968) with group velocity:

$$v_g = 2\bar{k}\times\left(\bar{\Omega}\times\bar{k}\right)/\left|\bar{k}\right|^3$$

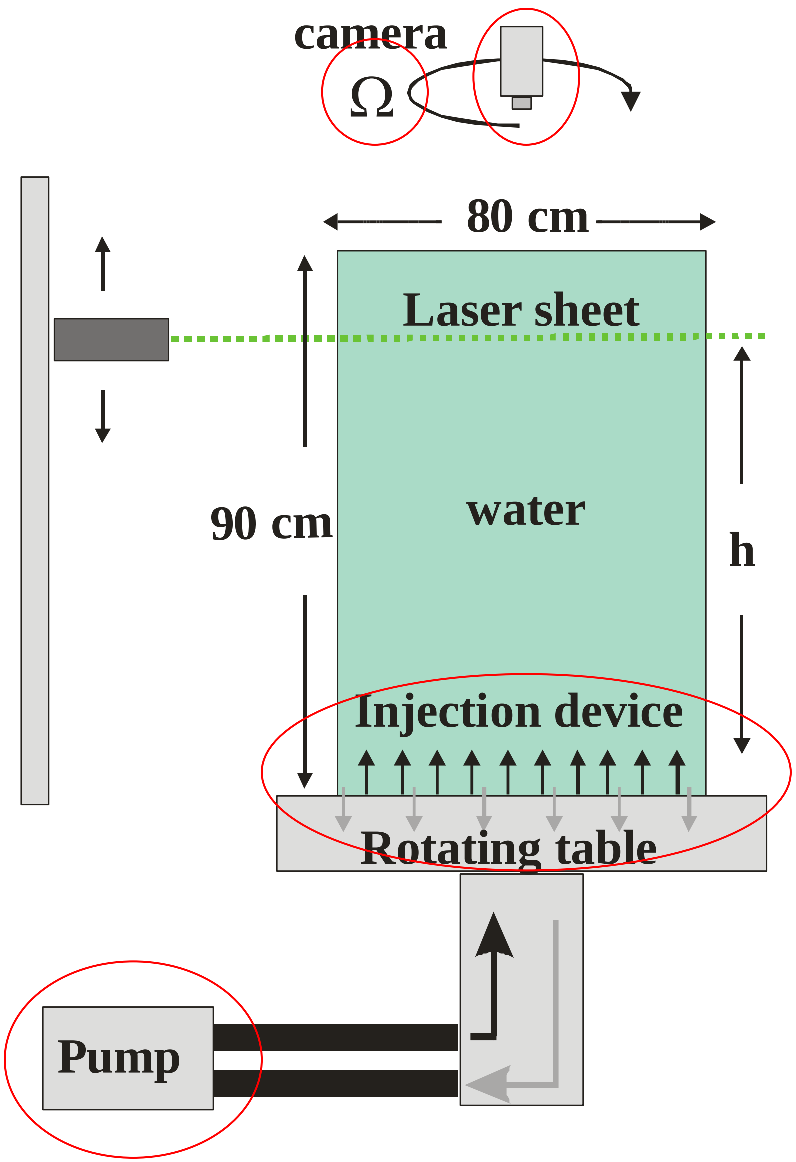
$$\boxed{\left|v_g(k)\right| = \frac{2\Omega}{|k|}}$$

The frequency of inertial waves must satisfy:

$$\boxed{\omega \le 2\Omega}$$

Are these waves important in turbulent rotating flow?

Experimental system

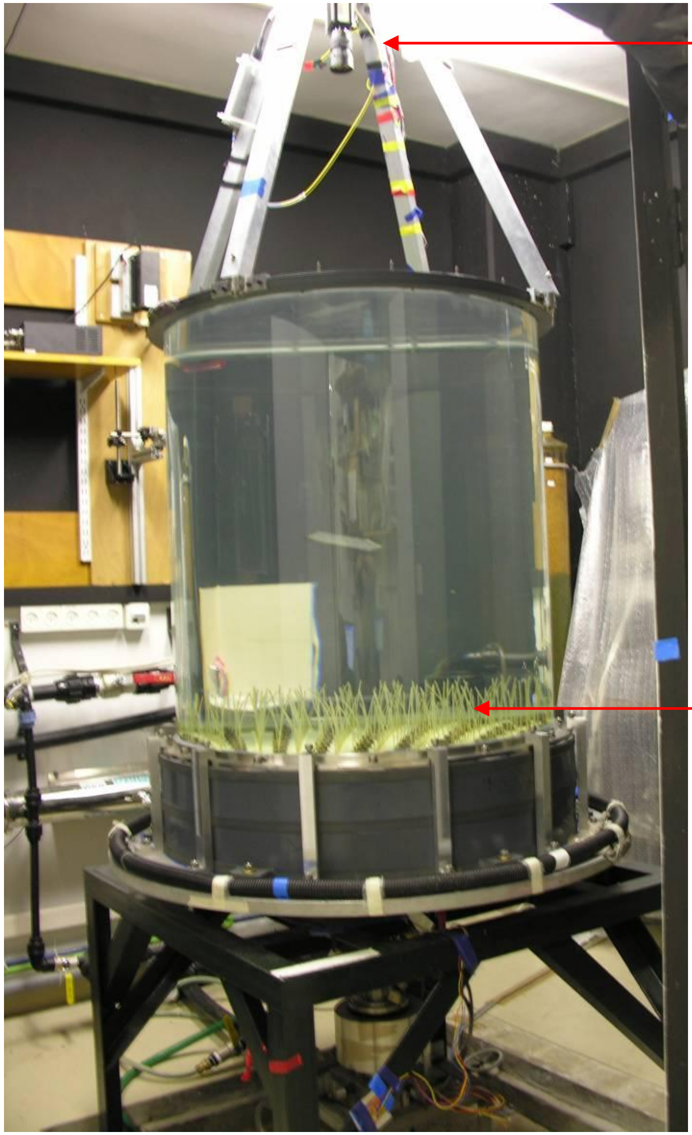


Ω up to 16 Rad/s

Max. flow rate 3 L/s => ~ 300 W

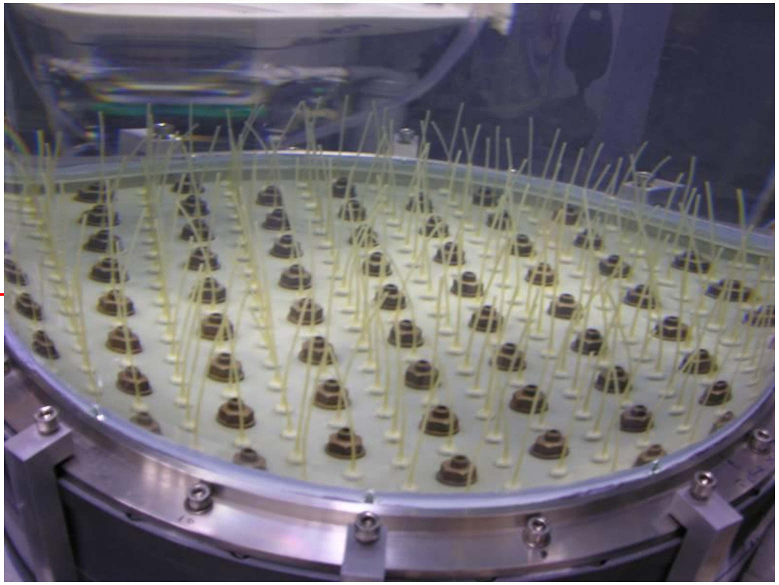
250 outlets and 70 inlets in hexagonal lattice

1 Mpix, 30 f/s, with less than 1ms "dead time", for PIV measurements

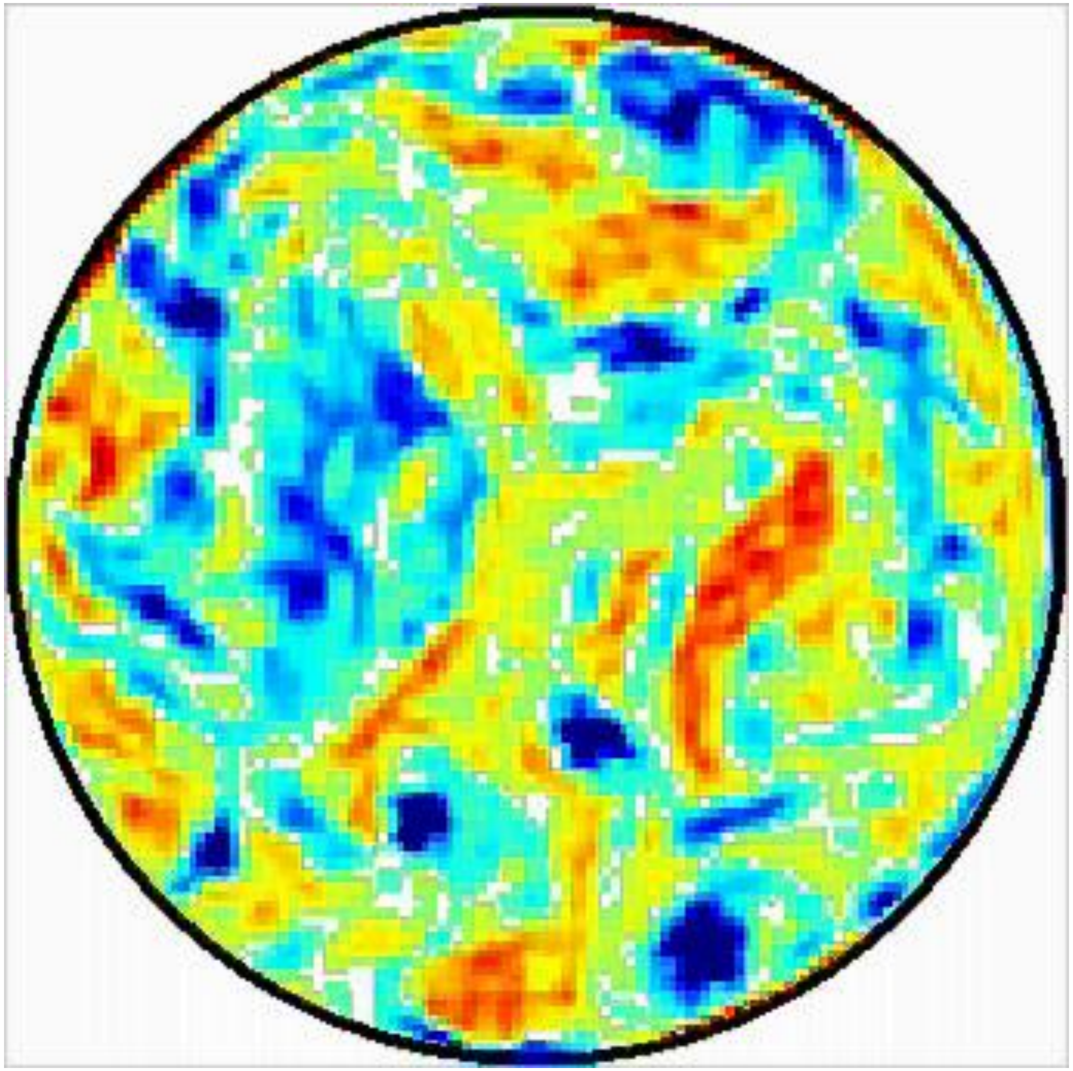


Camera

Injection Nozzles



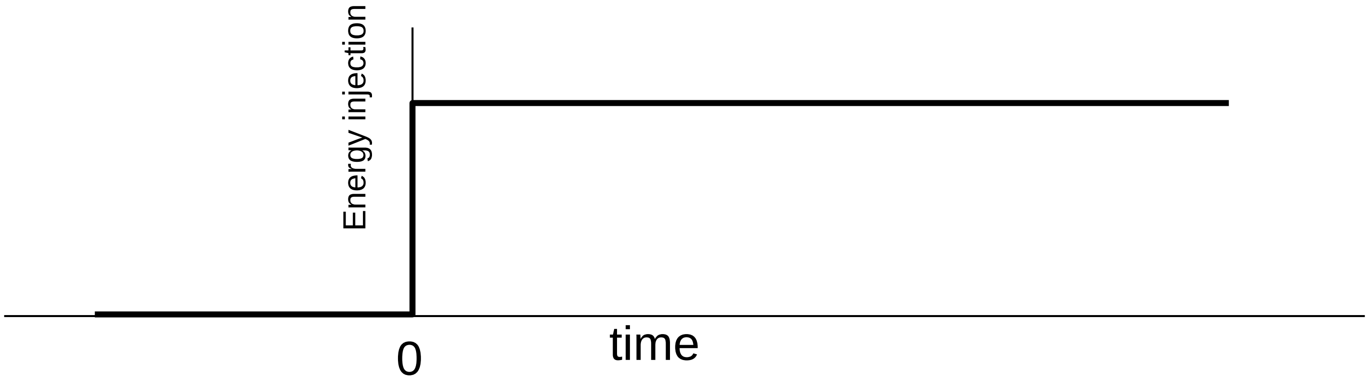
In a steady state (vorticity field)



Experiment 1

The system is brought to a solid body rotation (u=0) at a given rotation rate Ω.

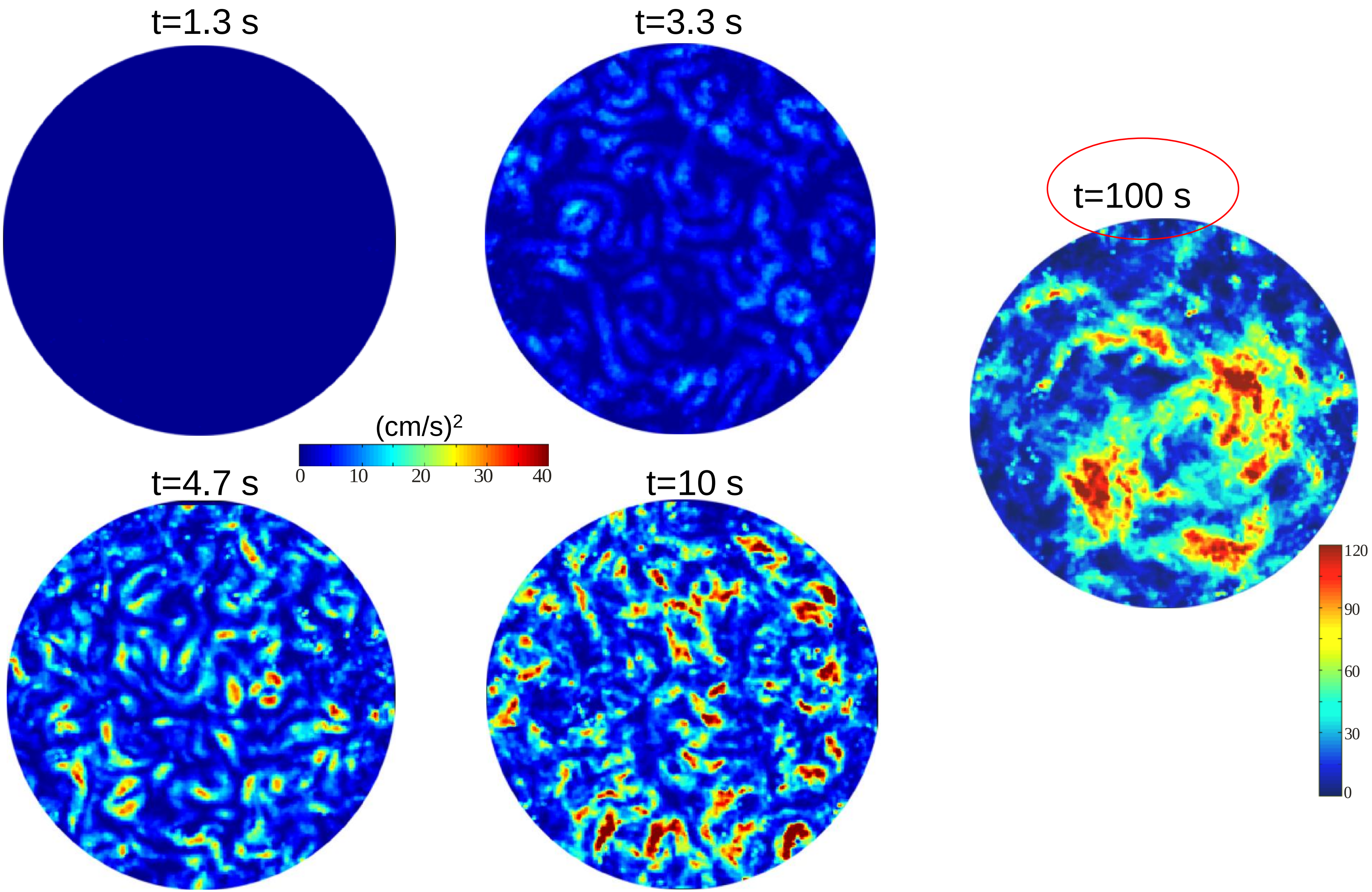
At t=0, we start injecting energy at a given flow rate (generating a step function in the injected power)



We measure the horizontal velocity (u,v) field at height H

Deriving energy power spectrum, E(k) and energy density “map”, (u^2+v^2), as functions of time.

Energy Density evolution

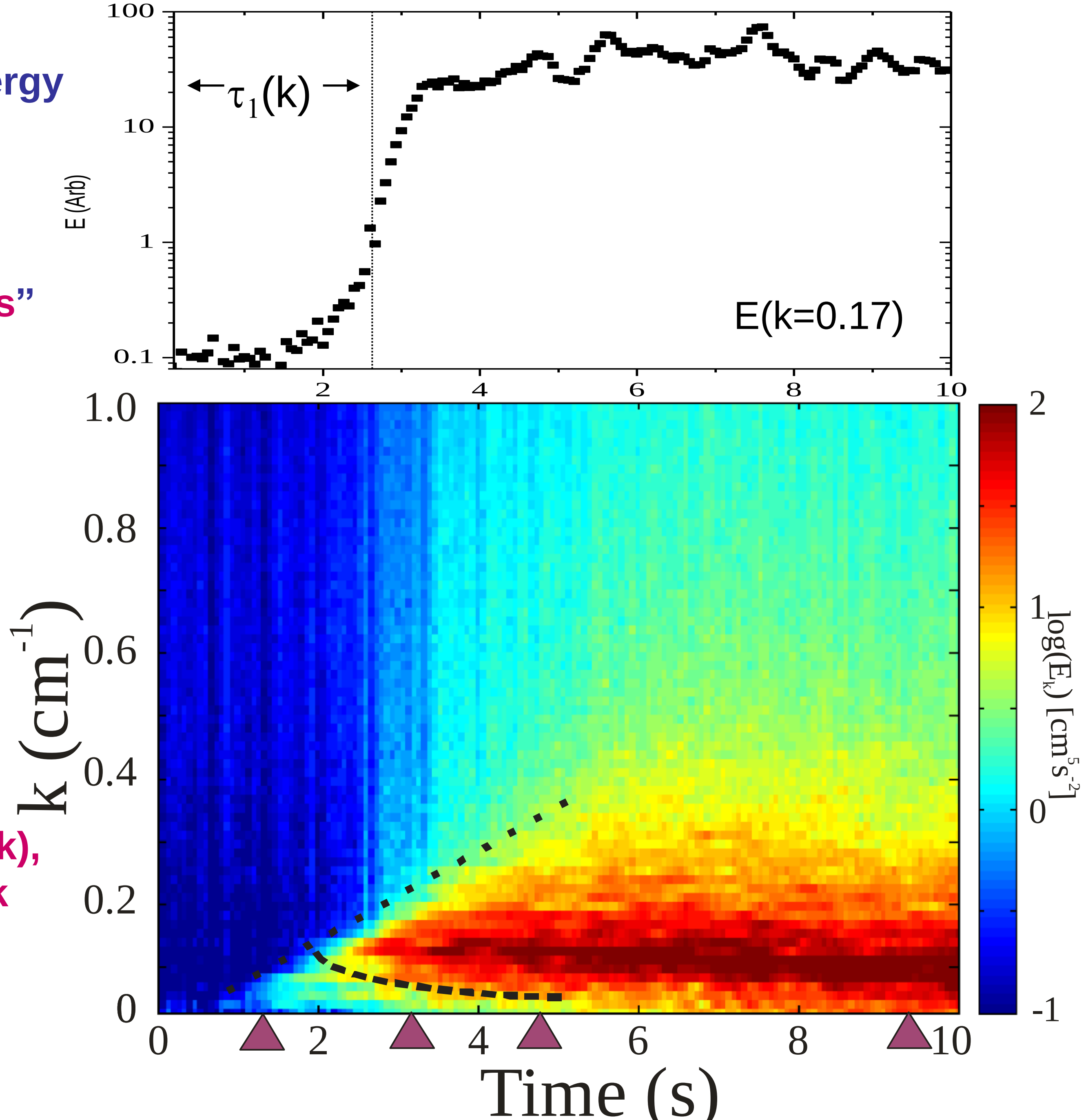


Sharp “arrival” of energy

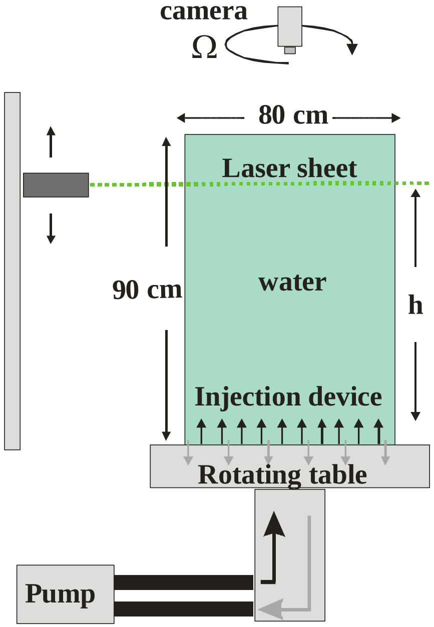
Two “Populating fronts” in the spectrum

One is linear in k, defining $\tau_1(k)$

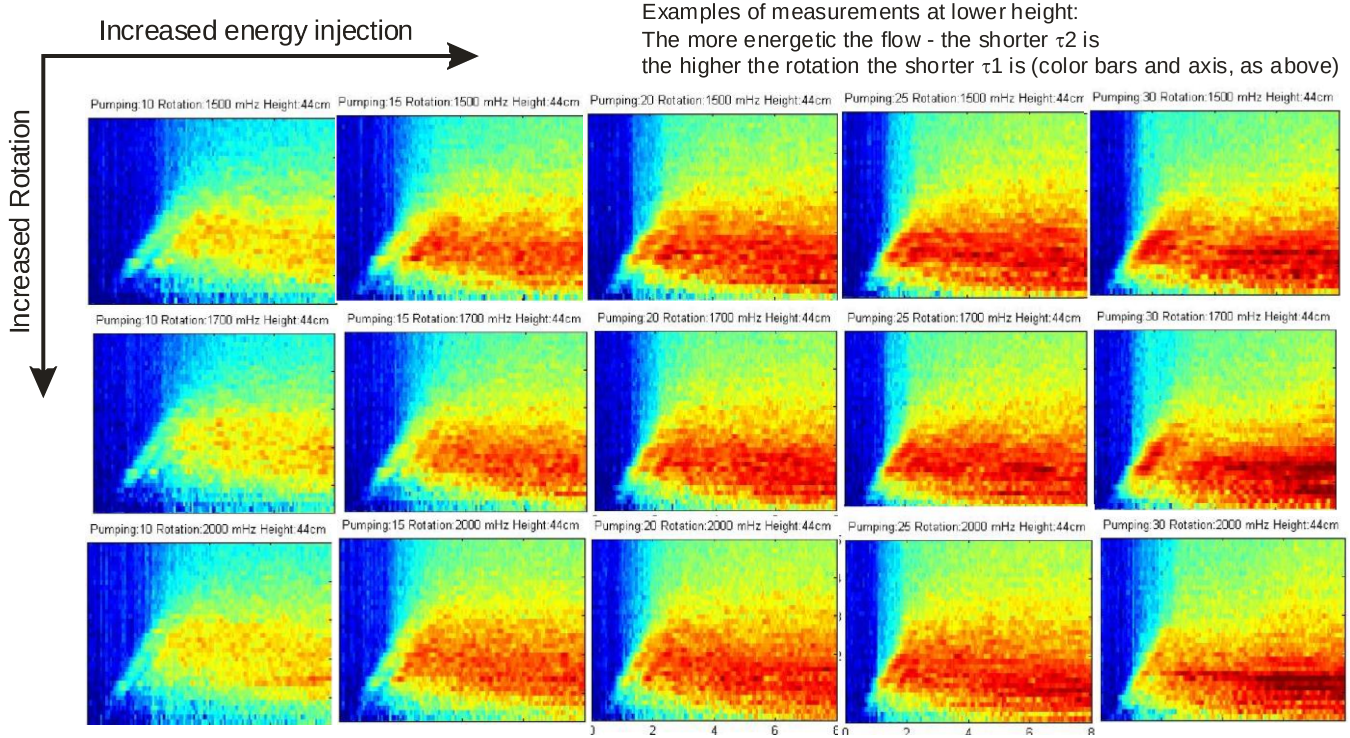
The second defines $\tau_2(k)$, which decreases with k

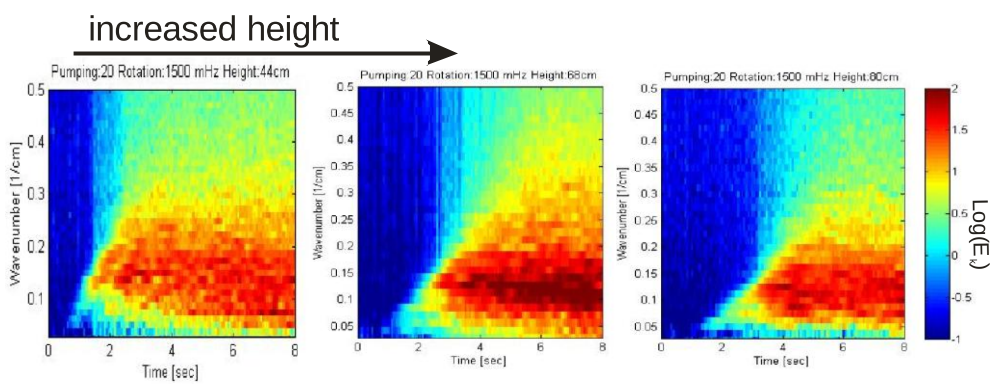


Variation of fronts properties with rotation, energy injection and height



Examples of measurements at lower height:
The more energetic the flow - the shorter $\tau 2$ is
the higher the rotation the shorter $\tau 1$ is (color bars and axis, as above)

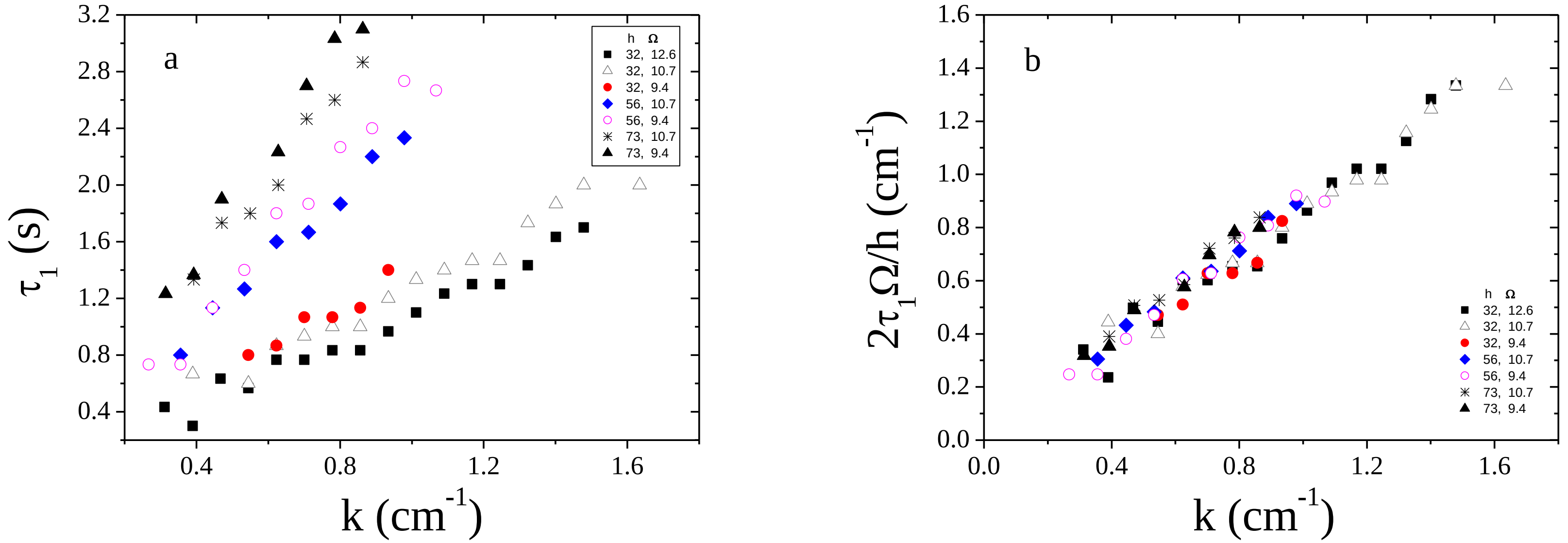




Scaling of τ_1

The energy carried by an inertial wave of wave number k would arrive at h by a time:

$$t = \frac{h}{v_g} = \frac{hk}{2\Omega}$$



τ_1 is the traveling time of inertial waves to the measuring plane

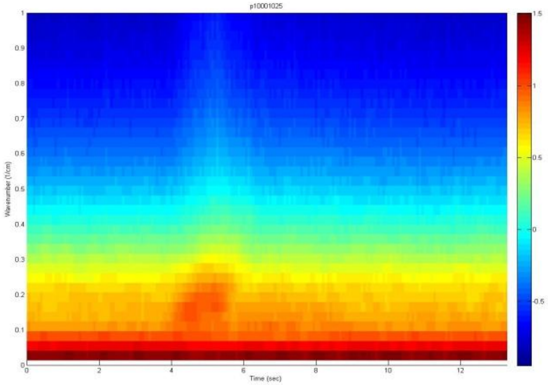
↓

All the energy transfer along z is done by inertial waves

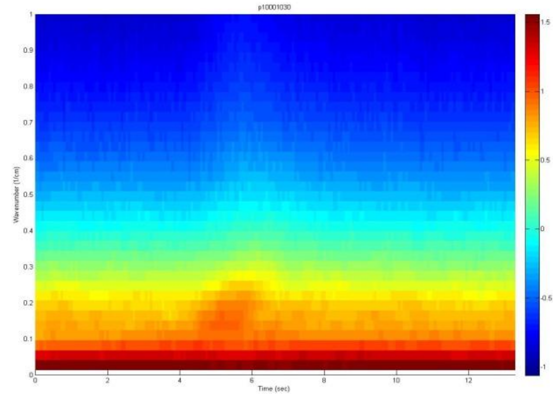
I. Kolvin, K. Cohen, Y. Vardi and E. S., *Phys. Rev. Lett*. 102**,** 014503,(2009).

Waves propagate within existing turbulence

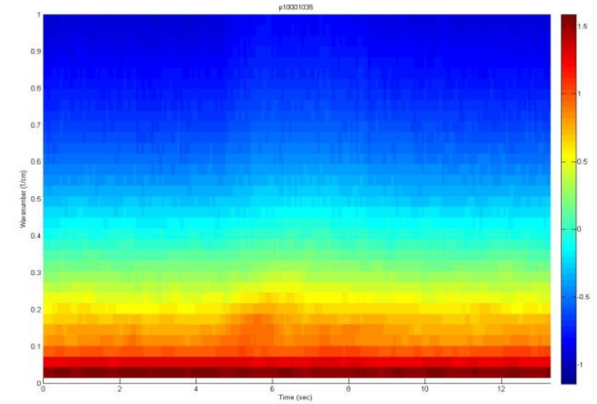
H=25 cm



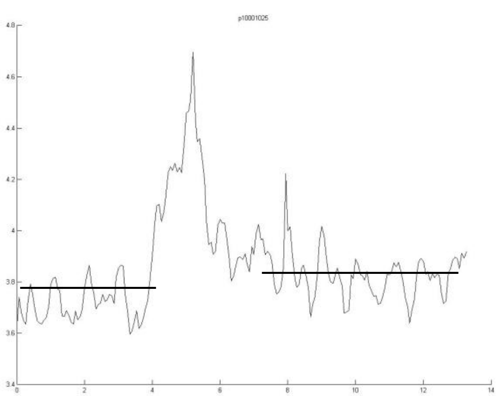
H=30 cm

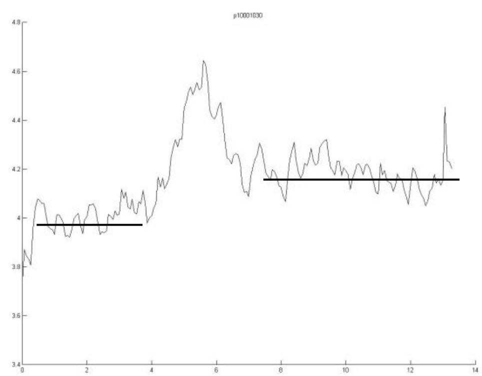


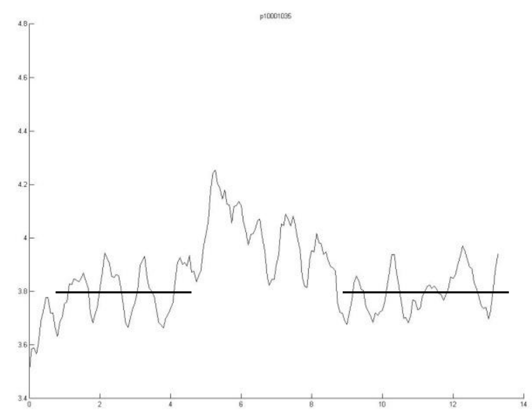
H=35 cm



Energy spectrum vs. time



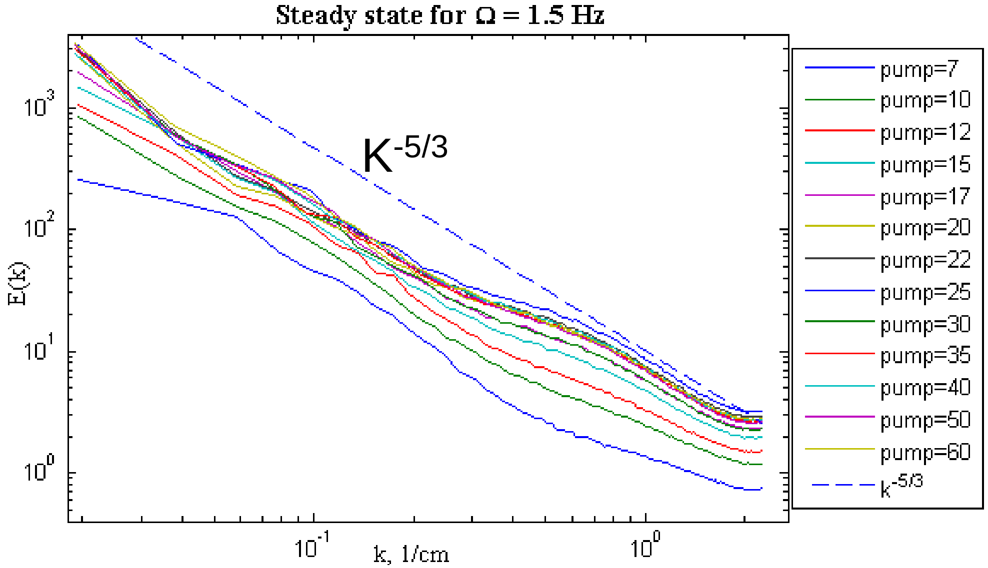


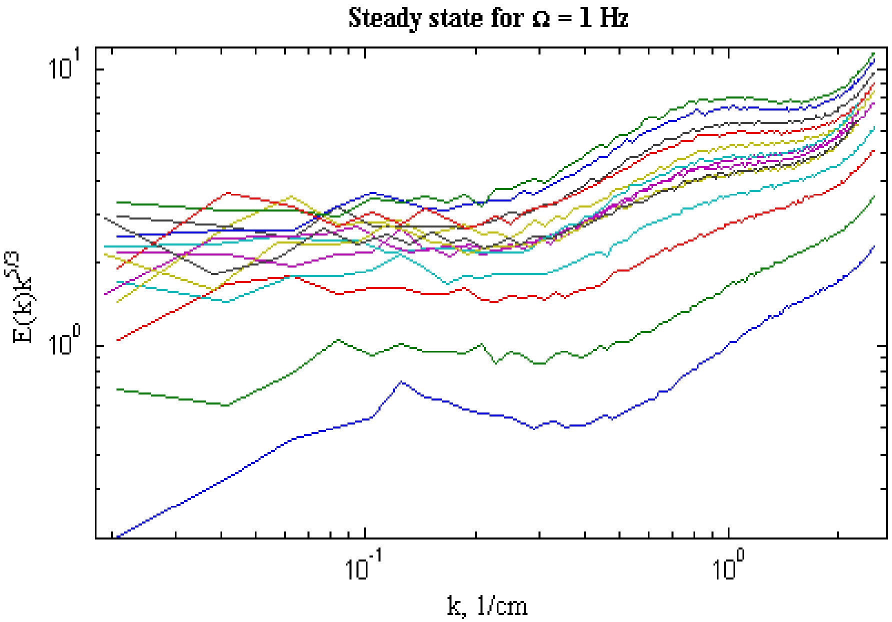


Total Energy vs. time

Are inertial waves significant in steady state?

Energy spectrum seems consistent with 2D turbulence

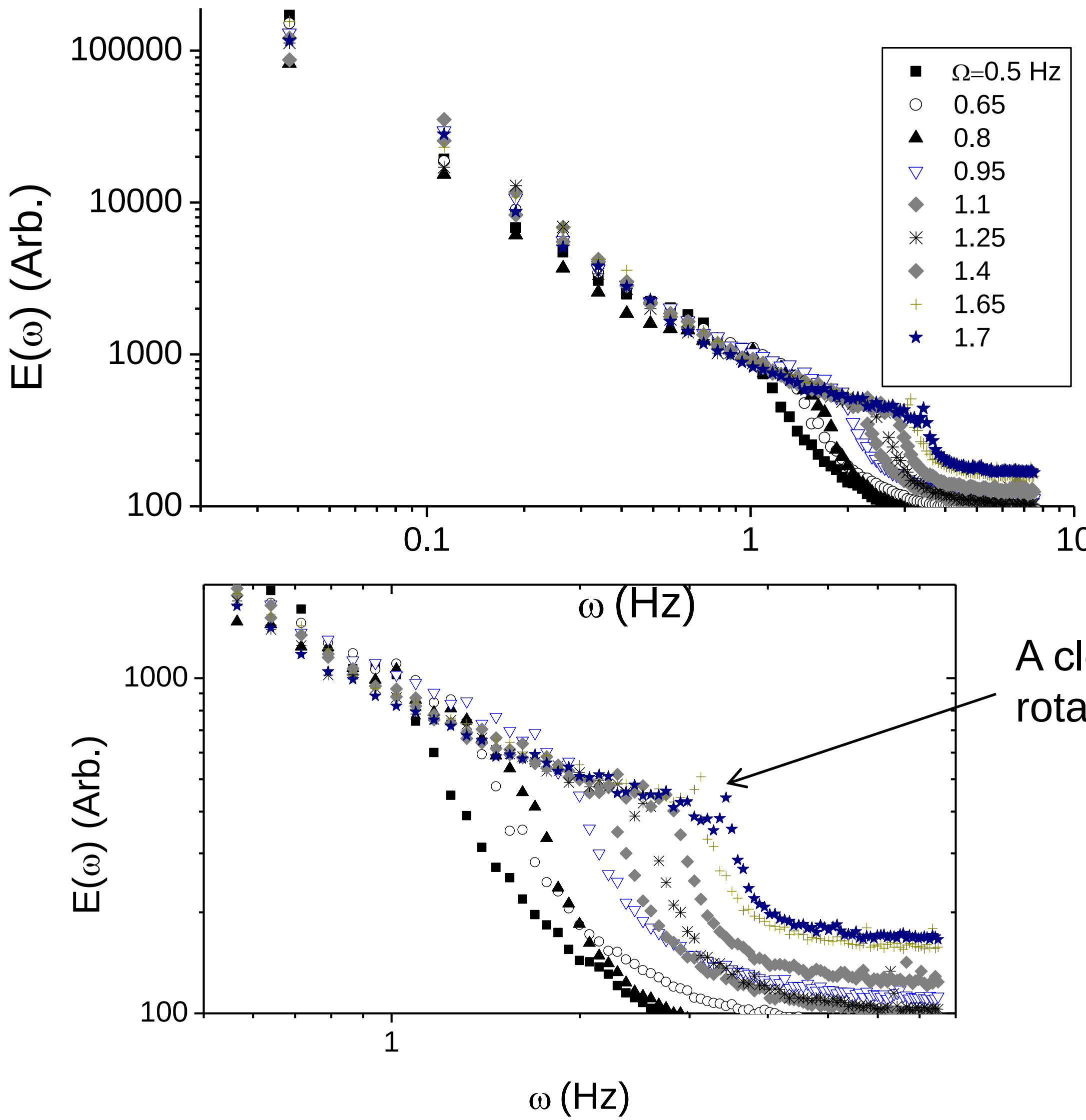




"compensated" spectrum

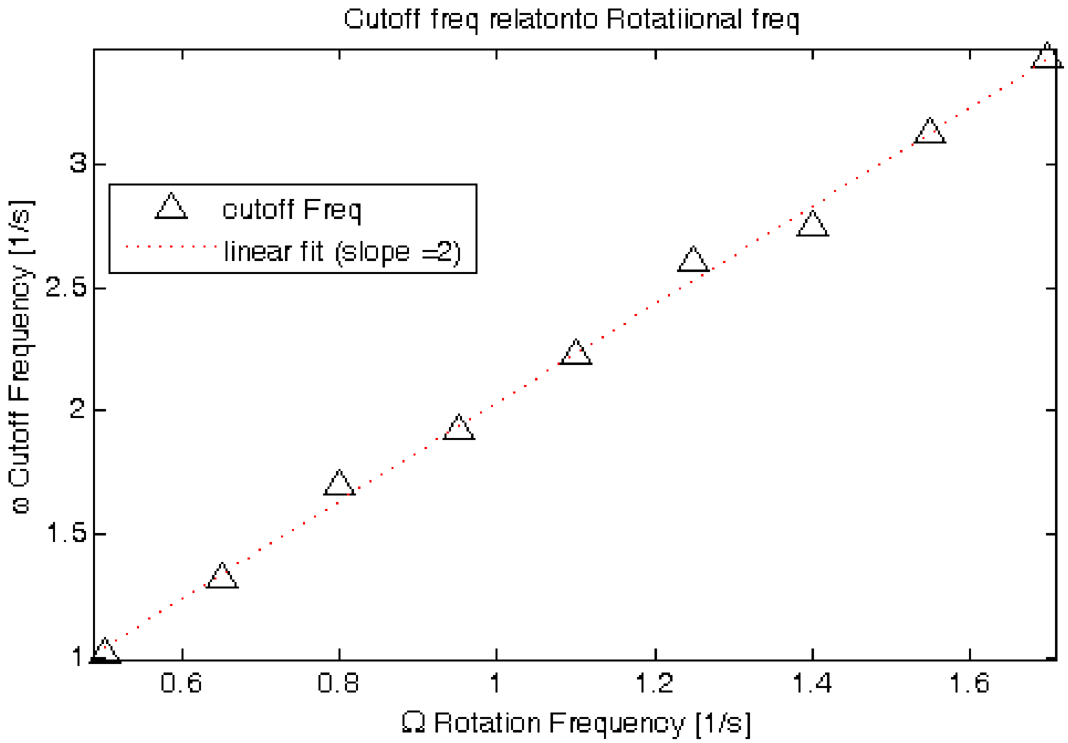
Measurements by Yuval Vardi

The temporal energy spectrum for different rotation rates



A clear “cutoff frequency for each rotation rate

The cutoff frequency vs. Ω



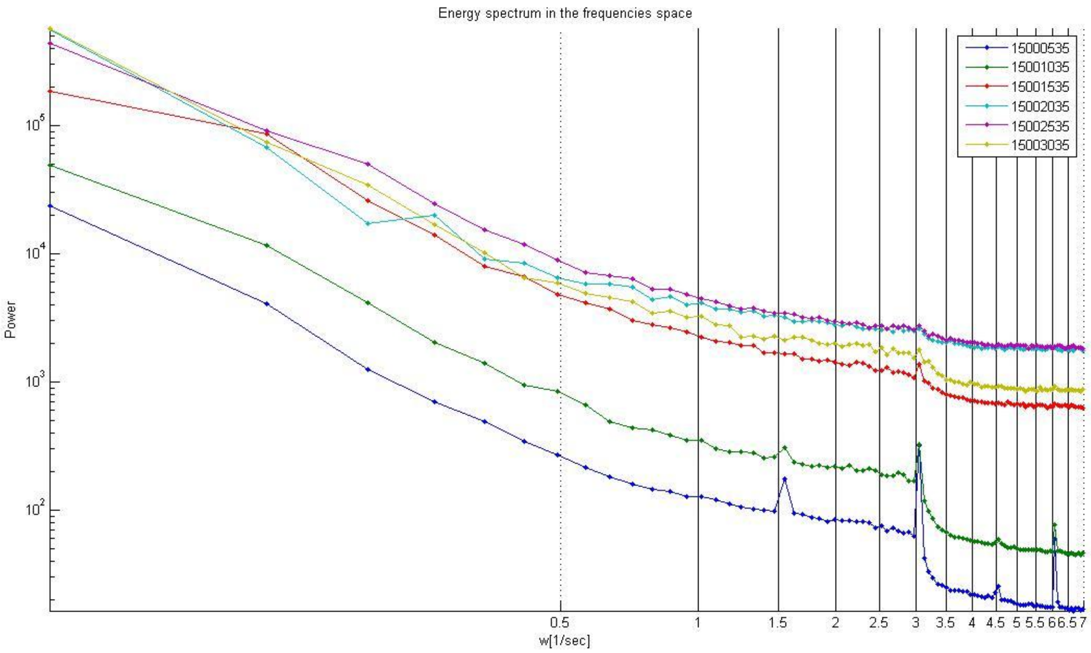
$\omega_{cutoff}=2\Omega$

Consistent with the cutoff in the dispersion relation of inertial waves:

$$\boxed{\omega \leq 2\Omega}$$

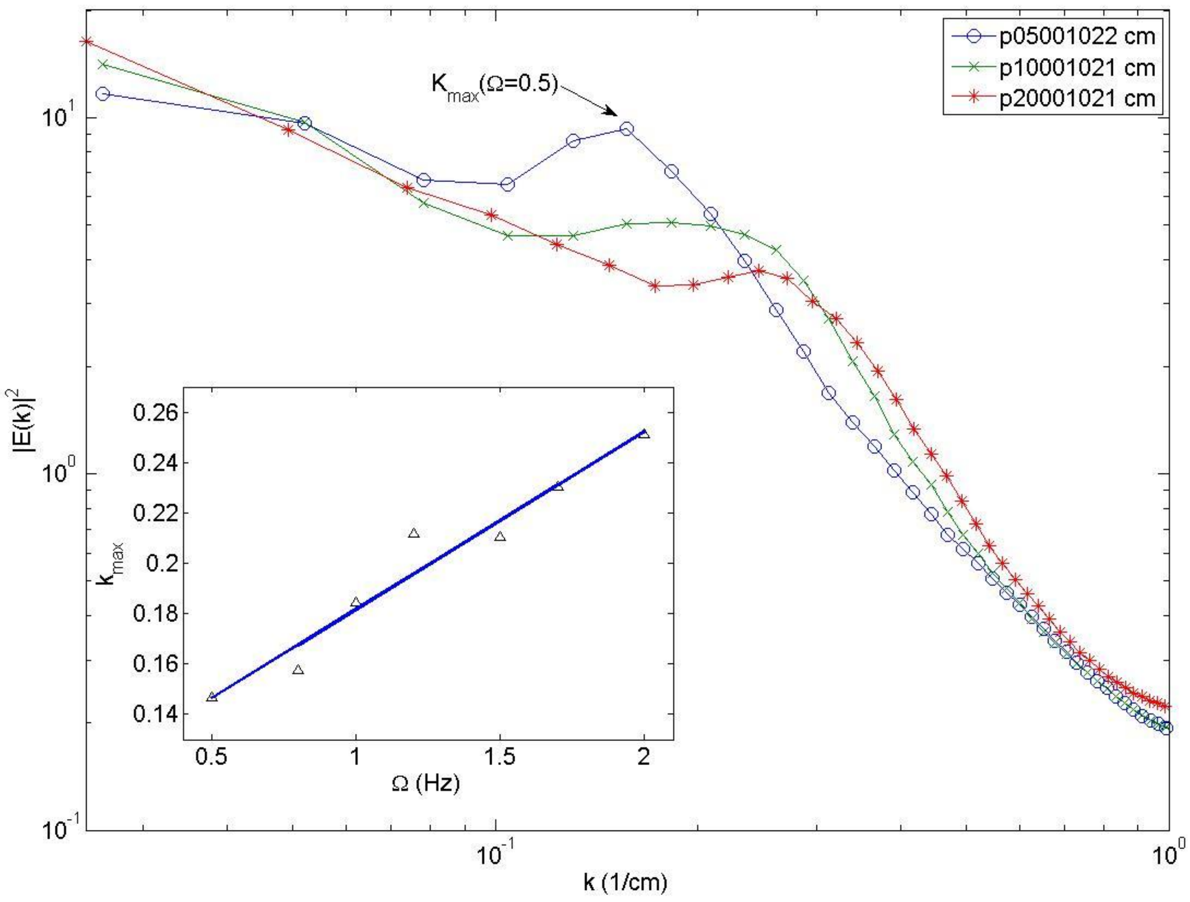
Suggests that inertial waves strongly affect the statistics even in a steady state

The temporal energy spectrum for different pumping rates



What is the dependence on the dissipation rate?

The injected energy spectrum at different rotations (same pumping rate)



A shift of k_{max} with Ω

Conclusions

High amplitude inertial wave packets can propagate in a turbulent rotating fluid.

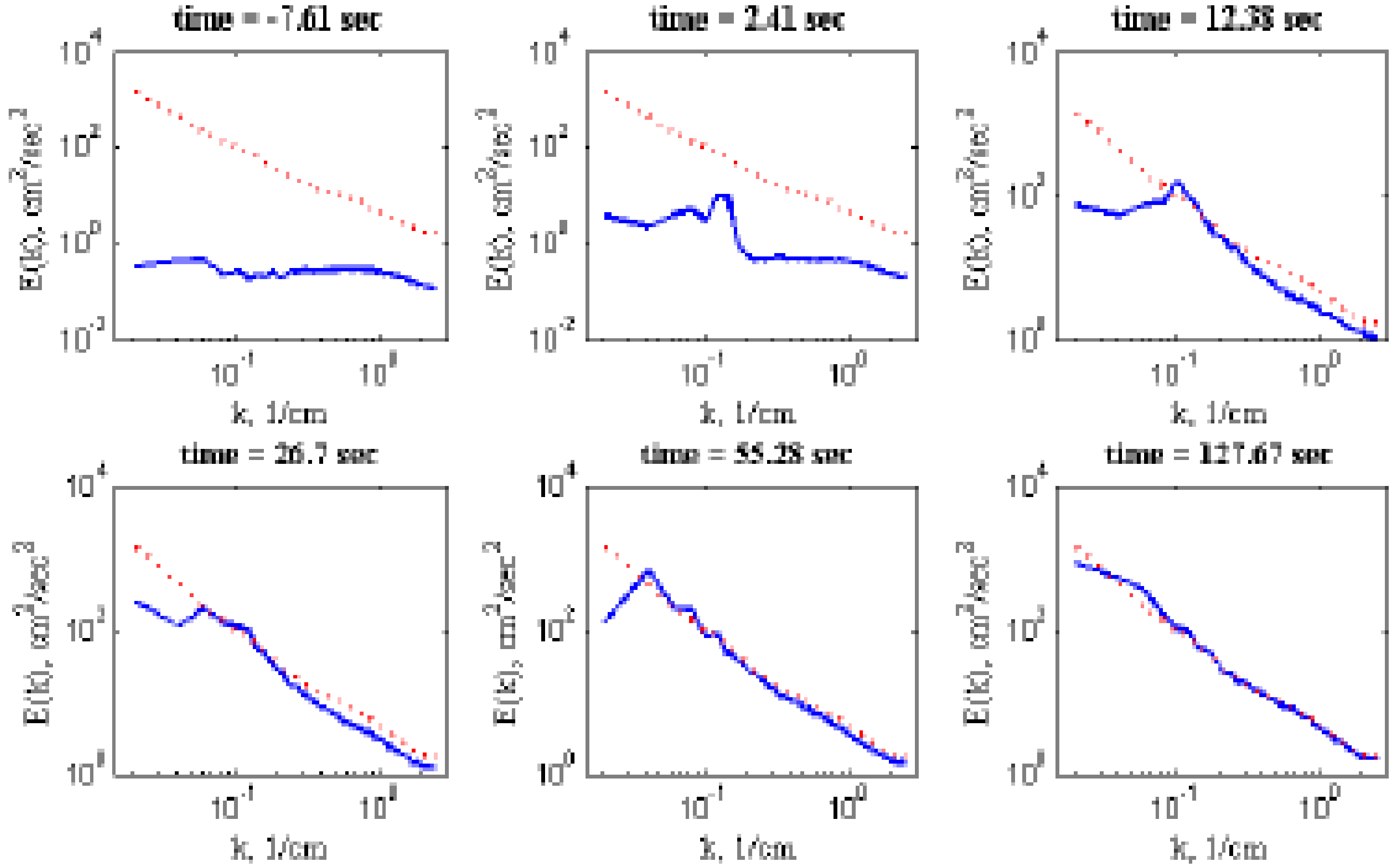
They propagate with the group velocity calculated for small perturbations

In the flow that was studied, energy is transported by inertial waves

Statistics of rotating turbulence, contains “finger prints” of inertial waves.

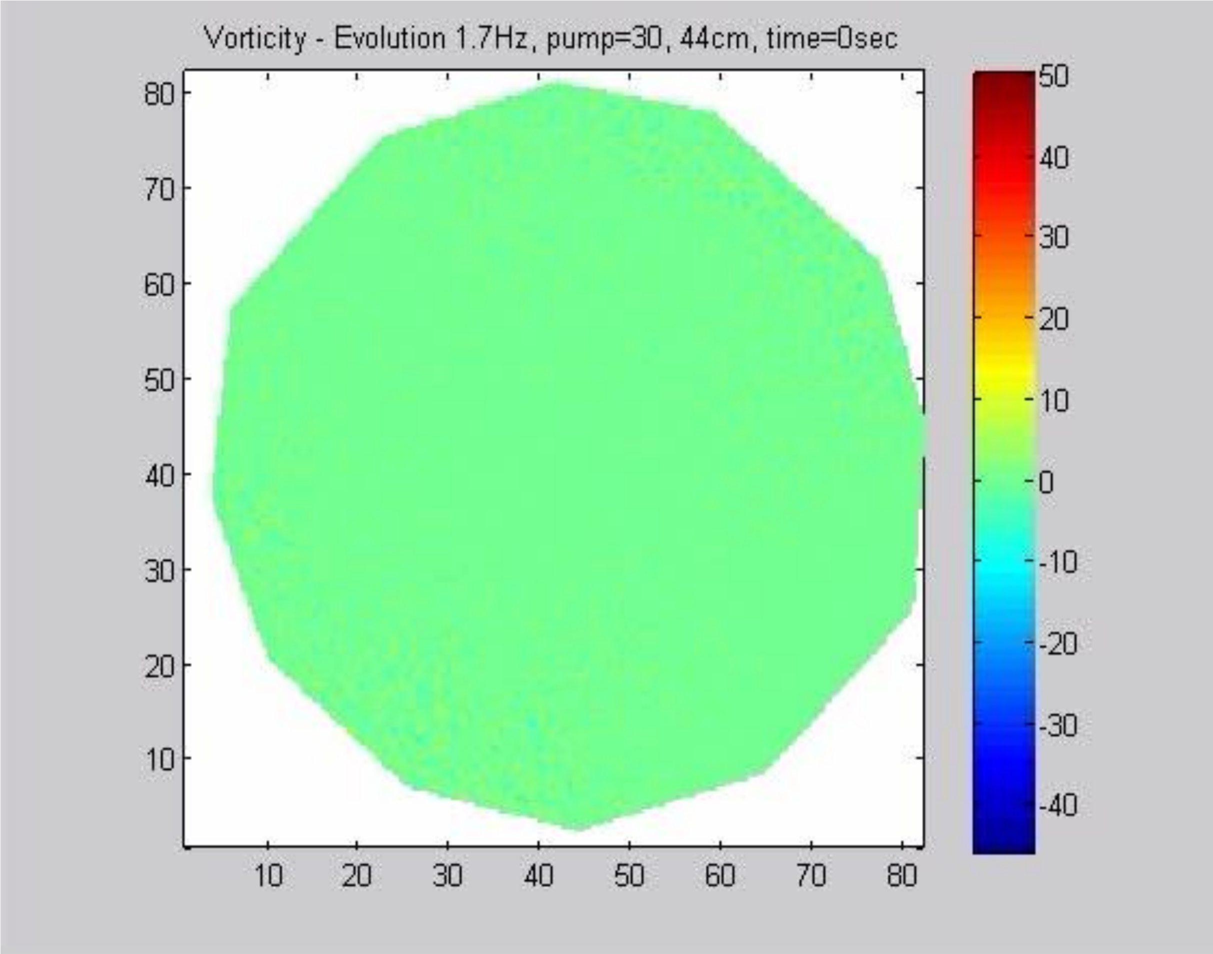
Can we write an “inertial wave turbulence” description?

Spectrum evolution over loner times

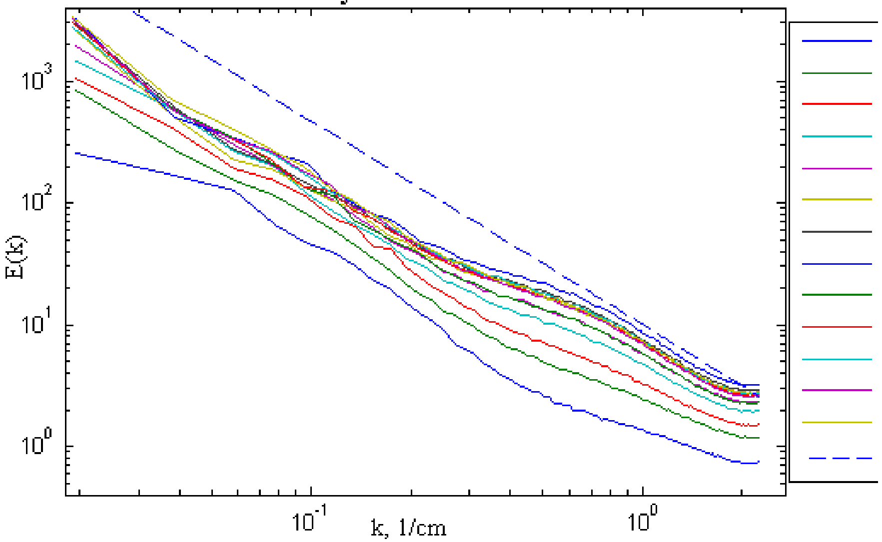


Vorticity

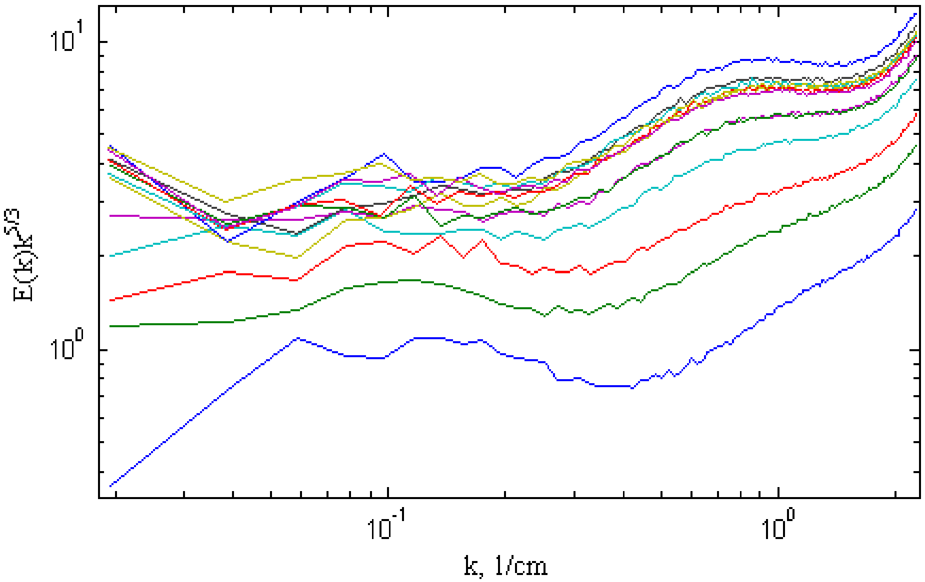
$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$



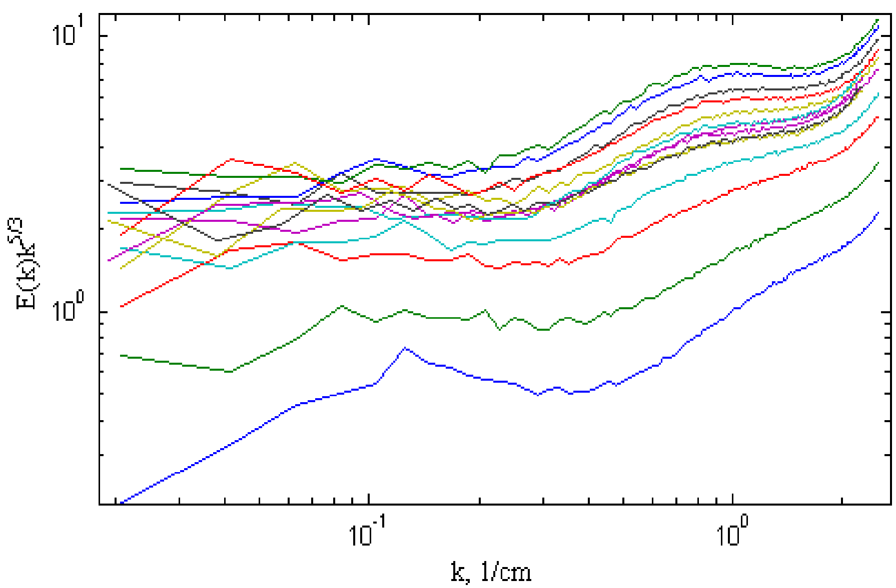
Steady state for Ω = 1.5 Hz

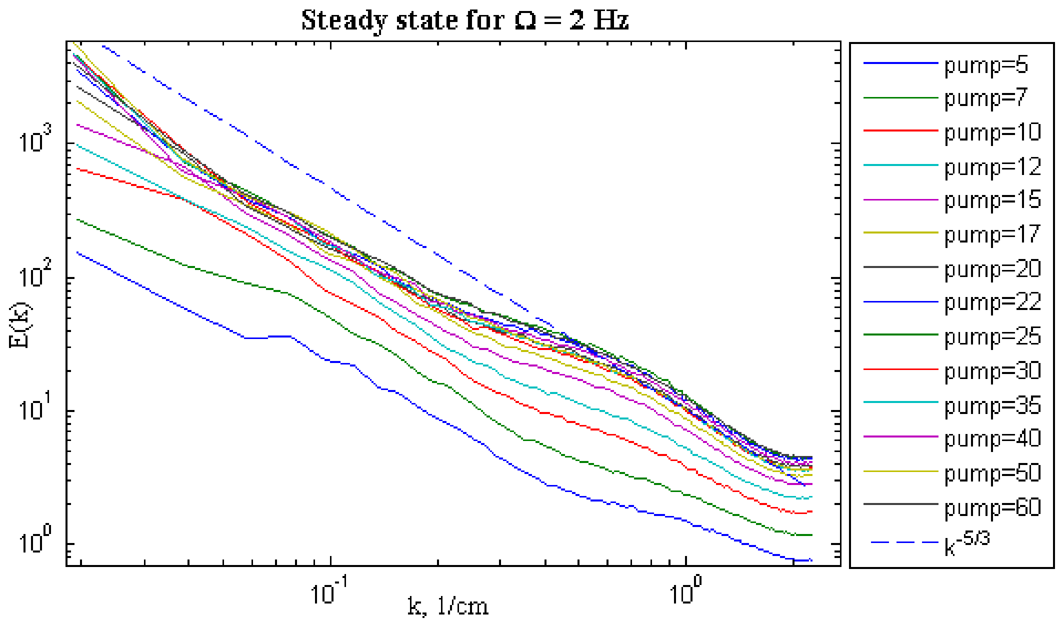


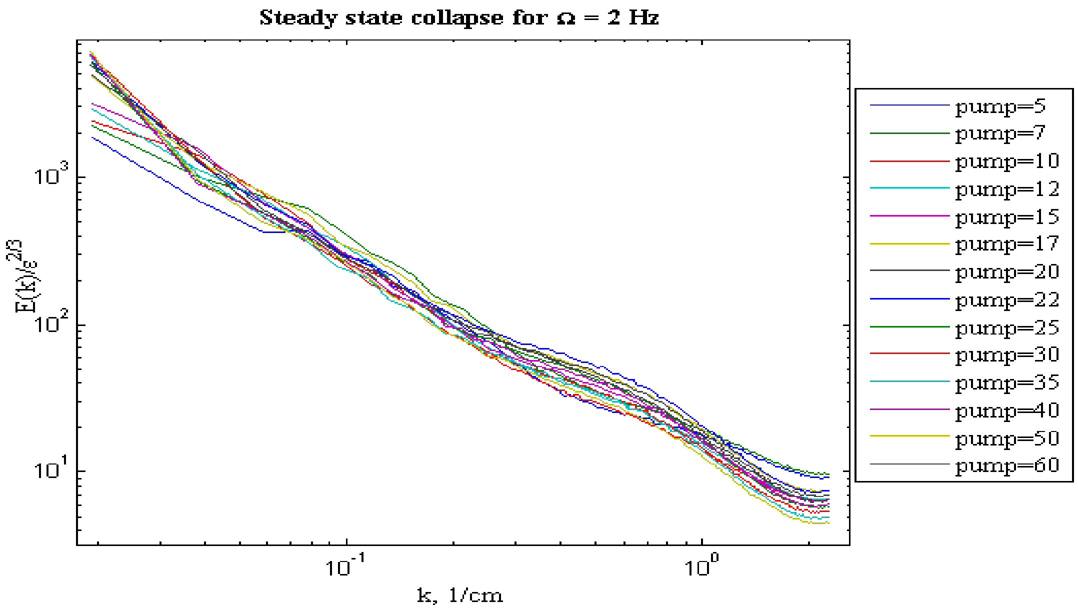
Steady state for Ω = 1.5 Hz



Steady state for Ω = 1 Hz







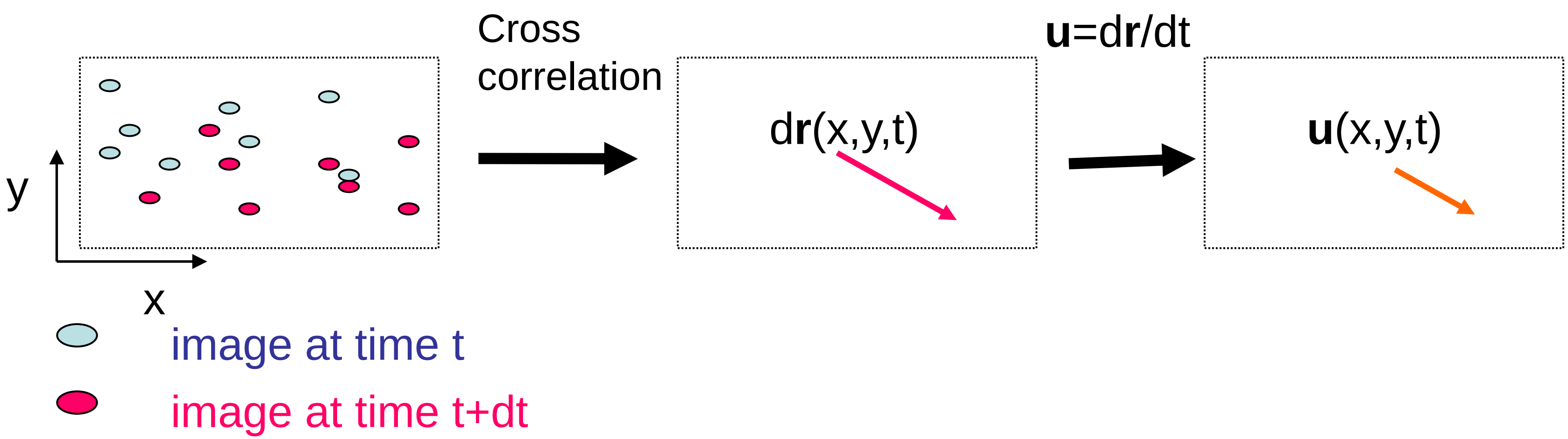
Scaling by $\varepsilon^{2/3}$

PIV system

•Using ~50 μm Polystyrene particles (ρ=1.06 g/cm^3) for light scattering

•Dividing an image to ~200* 200 “cells”, each with ~10 particles

•Calculating the cross correlation between images of different times, t and t+dt, within each cell



•Image 1 Megapixel, 10 bit
•Shortest dt ~ 1 ms (by strobbing the laser)
•$\mathbf{u}_{Max}$ ~ 1 m/s

Suggested scenario

$\tau_1 \sim Hk/\Omega$ – linear propagation time

$\tau_2 \sim (vk)^{-1}$ – nonlinearities of inertial waves

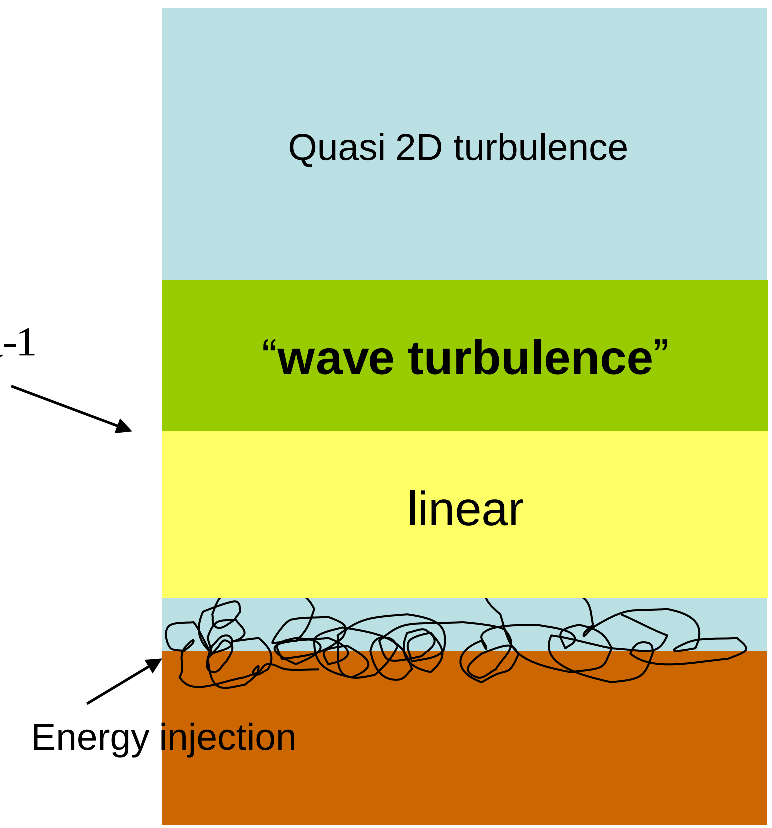
$\tau_3 \sim ?$ – time to achieve steady state

We have dominance of linear behavior for

$$k < \left(\frac{\Omega}{H\mathrm{v}}\right)^{1/2}$$

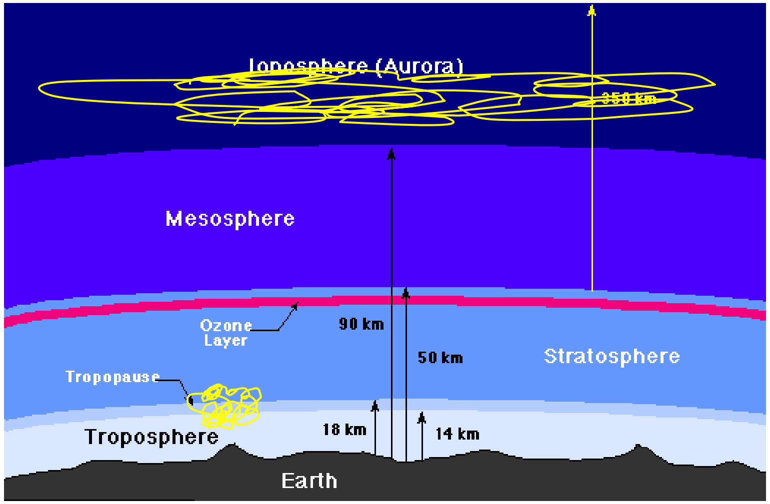
Or, for heights smaller than $H^* \sim \Omega k^{-2} \mathrm{v}^{-1}$

Above this height we expect to find a region dominated by (resonant?) nonlinear interaction of inertial waves – "wave turbulence"?



“Shooting” turbulence

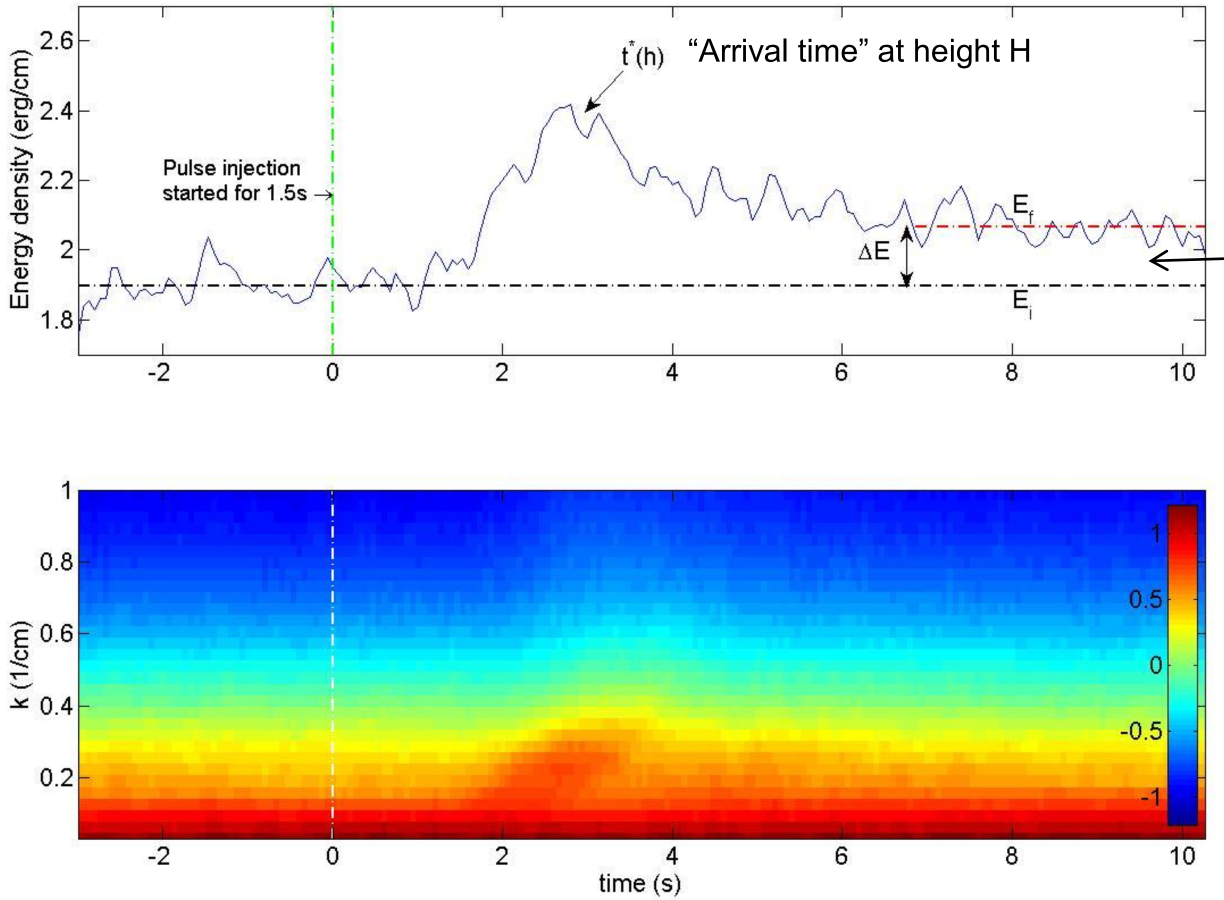
What happens in a turbulent atmosphere when there is a “sudden” blast of energy from a source?



Intuitively we would guess that most of the energy will accumulate near the source

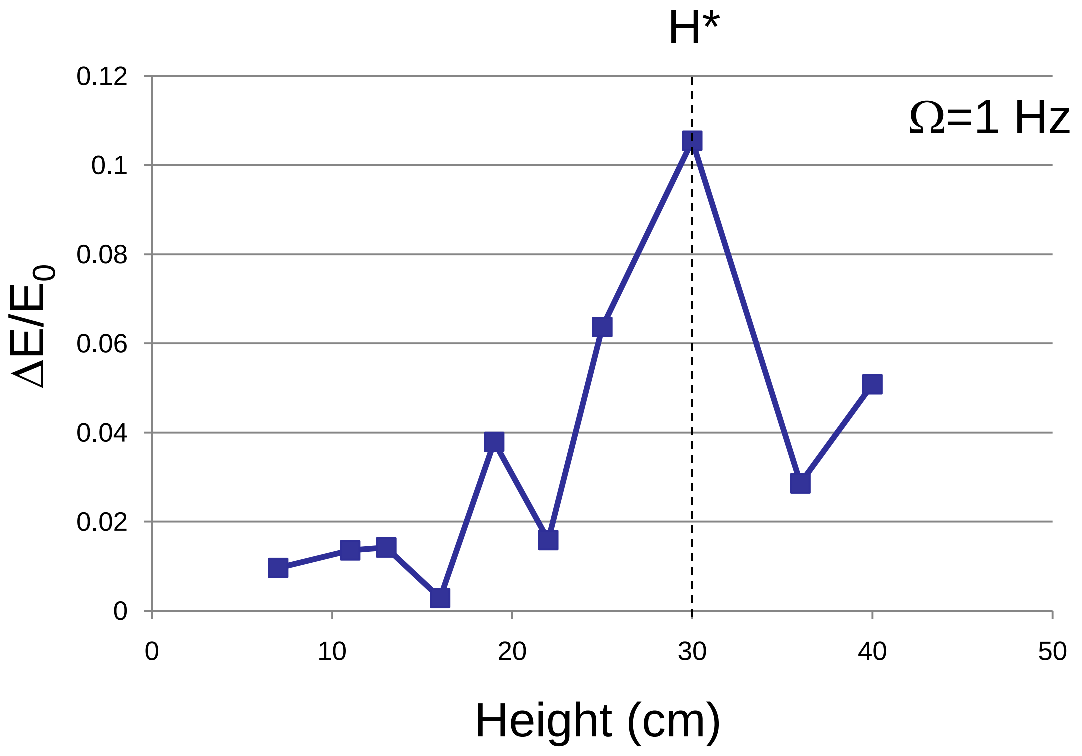
However, now we can predict, that the energy will propagate linearly upwards and will be transferred to the turbulent field only at height larger than H*

Measuring the propagating pulse



The energy that is “left behind” at height H

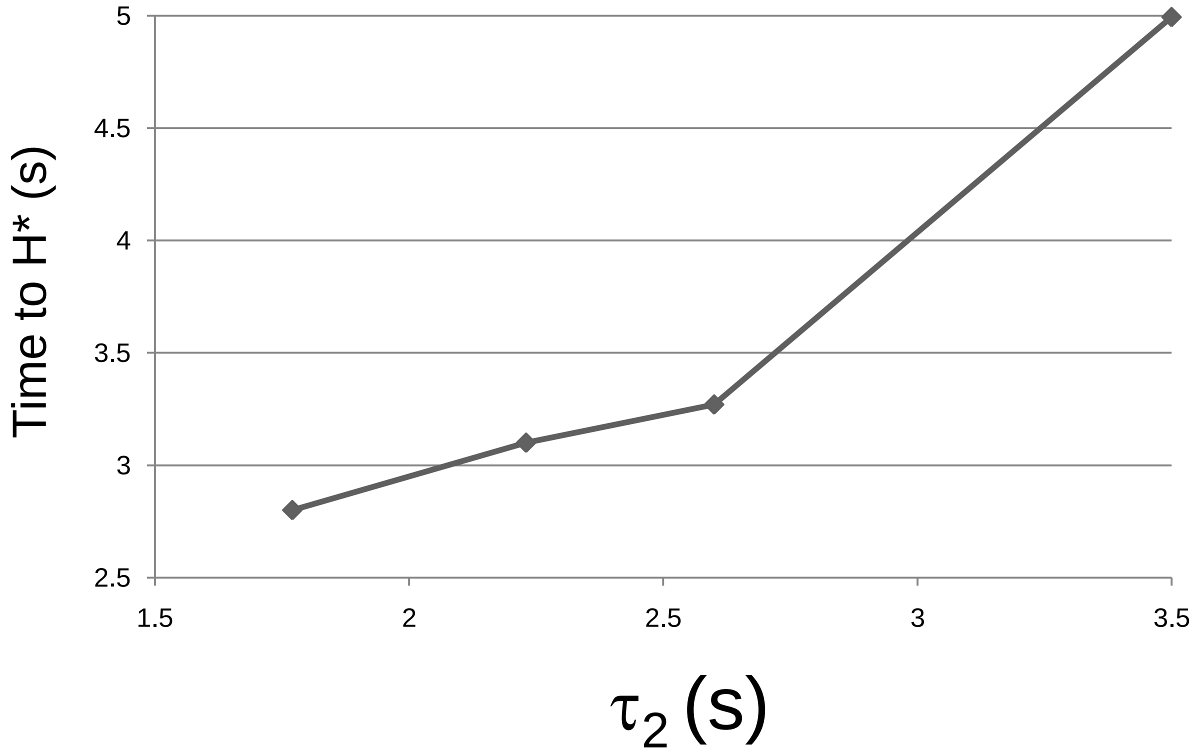
The energy “left behind” by the pulse vs. height



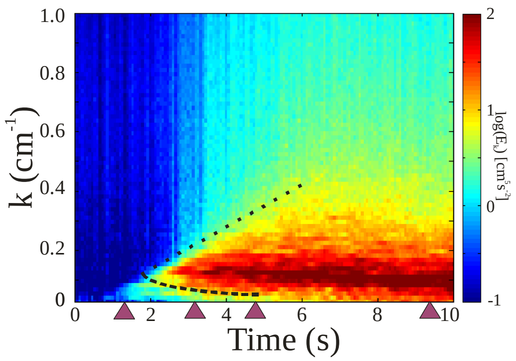
The energy of the pulse is concentrated at a selected height H*

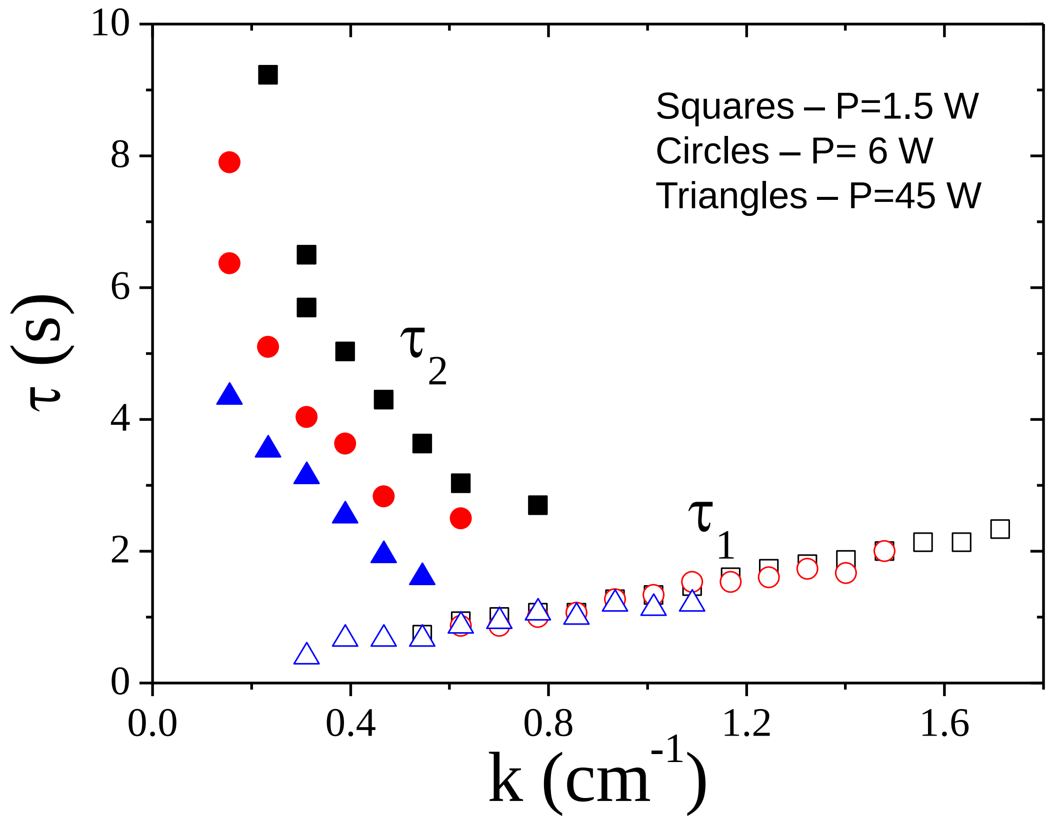
H* is determined by τ_2

Find the traveling time to H* under different conditions



Dependence of τ_1 and τ_2 on injected Power - P

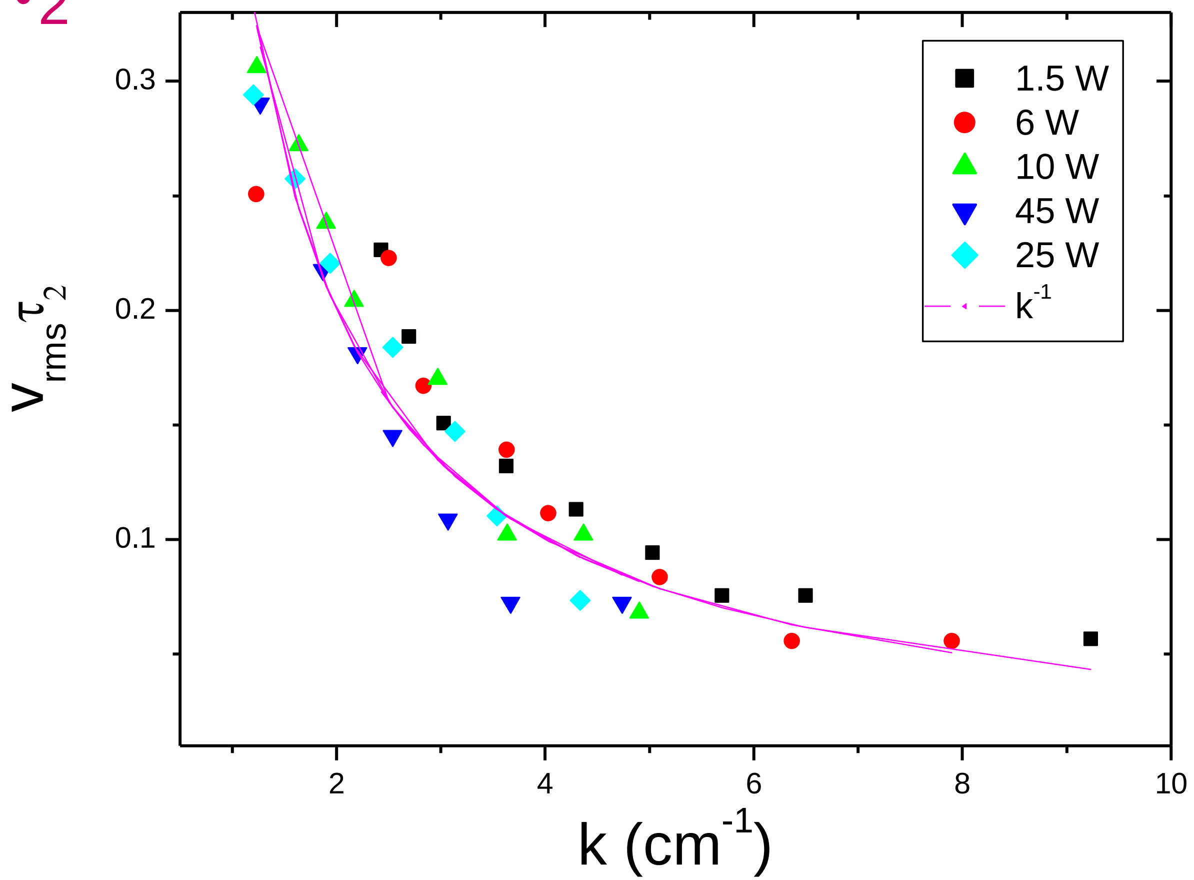




τ_1 is independent of P (as expected from a linear mechanism)

τ_2 decreases with P (An indication of the nonlinear nature of the underlying process)

Scaling of τ_2

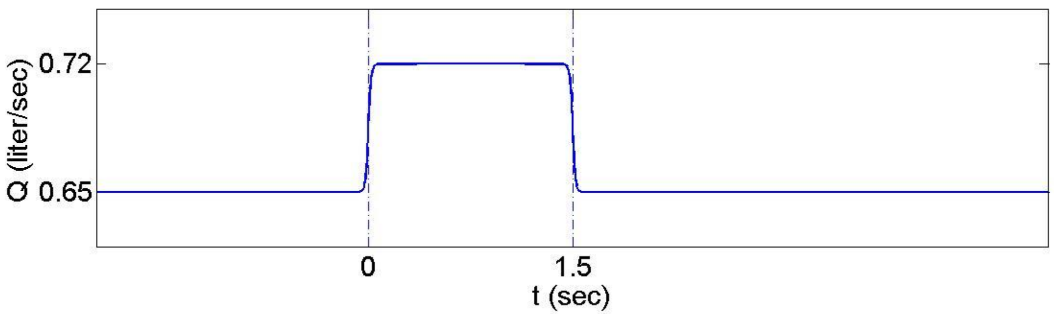


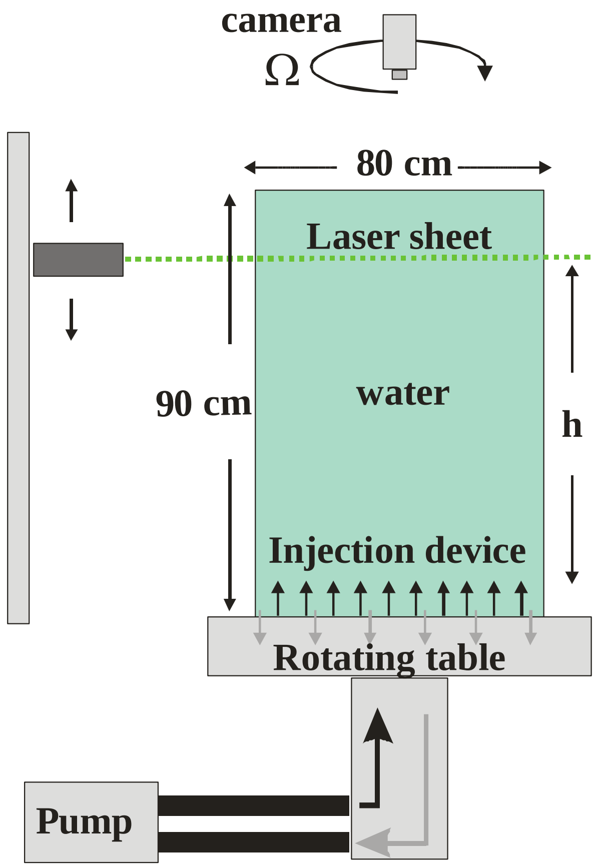
τ_2 scales roughly as the "eddy turnover time" - (vk)$^{-1}$

Should remember that steady state is achieved at times much longer than τ_2– is there a third time scale?

The experiment

Injecting a pulse of energy into a steady state turbulence field at H=t=0
And measure the velocity field at H>0.

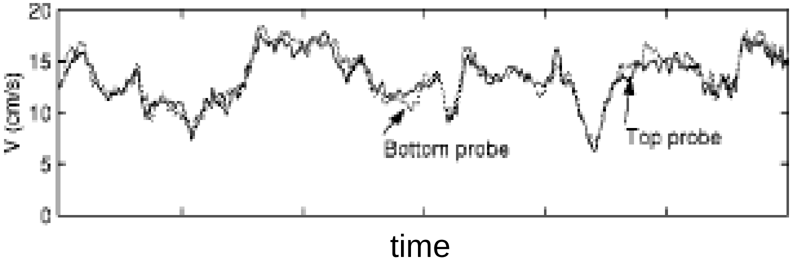




It is known:

Built up of large scales

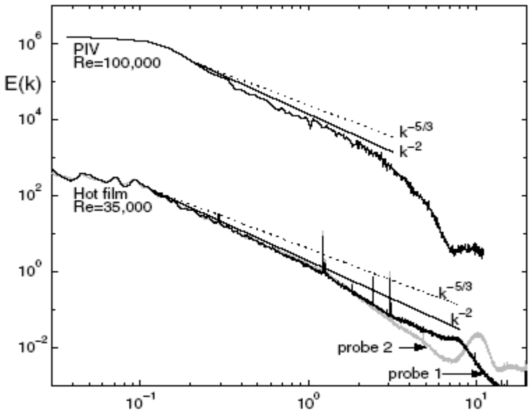
2D in the large scales (Baroud, Plapp and Swinney, 2003)



But also:

Different energy power spectra:

Baroud et. al. 2002



Smith and Waleffe 1999