The geometry of dissipative response in dephasing open systems

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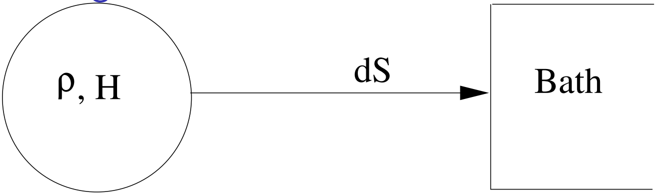
Dephasing bath



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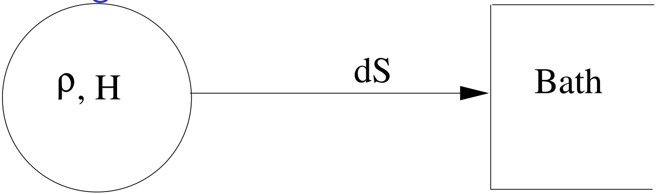

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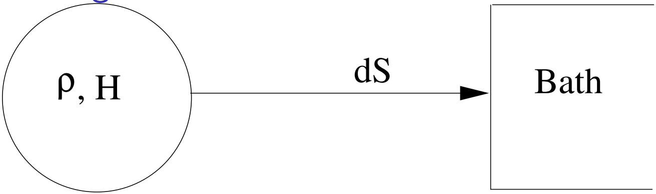

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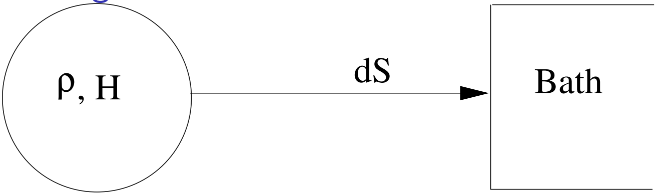

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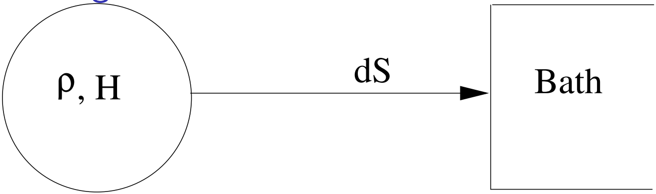

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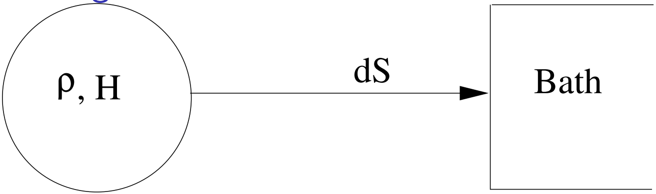

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$$\Gamma = \gamma^{1/2}\sqrt{H} \quad \text{or} \quad \Gamma \sim H \quad \text{or} \quad \Gamma_\alpha \sim P_\alpha.$$

Some References

Where one finds dephasing?

- NMR, cold atoms

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- P. Facchi, S. Pascazio: Zeno effect

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Why to study it?

- Family on the halfway between Hamiltonian dynamics and general open system dynamics. All energy eigenstates remain stationary,

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- Family to describe a dissipative response

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- Linear response of the observable F

$$\mathrm{Tr}(\rho_t F) = f\dot{\phi} + \cdots ,$$

here f is a response coefficient (e.g. conductance).

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Adiabatic theorem gives transition probabilities:

$$T_{0n} = -\frac{\gamma}{1+\gamma^2}\frac{1}{E_n - E_0}\mathrm{Tr}(\dot{P}_n(\phi_t)\dot{P}_0(\phi_t)) + O(\dot{\phi}^3).$$

Conclusion

The dissipative response coefficient

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Thank for your attention